Intersubjectivity and Groupwork in School Mathematics: Examining Year 7 Students' Interactions from a Perspective of Communicative Action

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Preface:

The schools and teachers involved in this study were part of a preliminary stage of research that led to the Raising Expectations and Attainment Levels of all Mathematics Students (REALMS) project funded by the Esmée Fairbairn Foundation. While the research was located within the context of this project, care was taken to separate the data and analysis in this thesis from the work done for the project. This being noted, some aspects of the work (in particular the task design work) described in this thesis informed analysis of the wider project and some of the data collection in this thesis was reported on to the wider research group in the course of regular research group meetings. All task development was in collaboration with participating teachers, and one of the tasks developed was done also in collaboration with another researcher from the same team (the Bracelets Task found in Appendix H). The transcripts of small group interactions and the teacher interviews and the associated analysis in this thesis are solely the work of the author.

Statement:

This thesis, whether in the same or different form, has not been previously submitted to this or any other University for a degree.

Geoffrey Kent

Signature: ____

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I am deeply grateful to the all the schools, teachers and students that I worked with over the past several years and their contributions to this thesis.

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Summary

This thesis explores how small group interactions around problem-solving in secondary school mathematics can be understood using a theoretical framework of Communicative Action inspired by Habermasian Critical Theory. How does cognition express itself socially? What are the technical features of communicative acts that afford access to the development of mutual understanding?

A case study approach was used to investigate episodes of interactive speech acts. Participants included three Year 7 mathematics teachers and 87 students in 3 different English secondary schools, who were engaged in adopting aspects of a 'Complex Instruction' pedagogical approach to design and coordinate problem-solving groupwork. Tasks were collaboratively designed with the participating teachers, followed by participant observation of the lessons, and post-lesson interviews with the teachers. Small group interactions were recorded using Flip cameras at each table that captured audio and video of student interactions around the tasks, and whole class video was also recorded. Initial analysis of small group interactions led to the development of codes and models focused on understanding interactions from an intersubjective perspective informed by Habermas' Theory of Communicative Action. These models and codes were then iteratively used to generate and refine analytical statements and working hypotheses from further interrogation of the data. The pragmatic focus of this study is on the content of episodes of utterances. These episodes are part of the intersubjective level at which teaching and learning take place. The findings from this analysis add to the field by developing a technical and critical treatment of evidence of intersubjectivity in mathematics education. Understanding the intersection of meaningful communication, action, and practices at the small group level is argued to provide novel insights into practice and design for problem-solving groupwork in mathematics education.

The contributions of this thesis include the development of an Intersubjective Framework for Analysis of small group interactions, evidence that this framework can be productively used to identify ways in which the development of collaborative understanding expresses itself at the small group level, how it breaks down and how it can be supported. Methodologically this work makes a claim to knowledge in the development of microanalyses of situated cognition informed by Habermasian social theory. This work explores the merits and limitations of the communicative perspective in understanding small group interactions in mathematics problem-solving situations. A central claim is that Habermas' sociological approach can be used productively to investigate small group interactions in mathematics classrooms.

Theoretically this work makes a claim to knowledge in the development of a novel set of codes and models that can be used to analyse evidence of intersubjectivity through analysis of episodes of utterances in situ. This analytical framework is used to argue that small group interactions can be understood productively from a theoretical perspective of Communicative Action. These contributions suggest that insights from a perspective of Communicative Action can give educators critical pragmatic insights into curriculum design, structuring groupwork and associated pedagogy, and communicative (as opposed to instrumental or strategic) intervention in the support of intersubjective understanding.

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Tables and Figures

List of Acronyms

AMS	American Mathematical Society
CME	Critical Mathematics Education
CSCL	Computer Supported Collaborative Learning
ESRC	The Economic and Social Research Council
GCD	Greatest Common Divisor
IMP	The Interactive Mathematics Program
IRE	Initiation-Response-Evaluation communication pattern
JRME	Journal for Research in Mathematics Education
LCM	Least Common Multiple
MMAP	The Middle School Mathematics Through Applications Project Group
NCTM	National Council of Teachers of Mathematics
NSF	The National Science Foundation
Ofsted	Office for Standards in Education, Children's Services and Skills
PGCE	Postgraduate Certificate in Education
QCA	Qualifications and Curriculum Authority
REALMS	Raising Expectations and Achievement Levels for all Mathematics Students
REF	Research Ethics Framework
SFOCC	San Francisco Office of Citizen Complaints
TA	Teaching Assistant
TCA	Theory of Communicative Action
TIMMS	Trends in International Mathematics and Science Study

Chapter 1: Introduction

1.1 Setting the scene for the research

In seeking to understand the form of communicative practices of students in small group interactions around mathematics learning, this thesis argues that a model of communicative action, based in Habermas' Theory of Communicative Action (TCA) (Habermas 1985a, 1985b), can inform the analysis of how students learn in the context of complex instruction style groupwork, why it may be effective, and how student learning and understanding of mathematics can be understood and supported. Further, this thesis argues for a critical potential of these technical models in the illumination of technical features of inequity, power and conflict at the level of interactive utterances. The critical potential of theories and tools developed in this thesis indicates how this work may serve as a lens for addressing issues of access, rates of participation, attainment, and wider issues of social justice in the analysis of small groupwork in mathematics education.

This thesis uses case study research to explore the interaction among students engaged in problem-solving within small groups in the context of the adoption of a particular form of non-traditional practice, Complex Instruction, in Year 7 heterogeneously grouped (mixed ability) mathematics classrooms in England. Analysis of collected data includes the development of a set of codes for utterances and a model of students' interactive groupwork. Utterances here are understood as the discrete speech acts made by participants, which realise their meaning in the episodes of interaction with others. The model of students' interactive groupwork is a theoretical framework to help make sense of how these utterances interact and relate to one another in communication. Working with three teachers at three different schools that were adopting complex instruction in unset year seven mathematics classes, I participated with teachers to develop 'group worthy' mathematics tasks and then engaged in participant observation of the lessons in which the tasks were used. I also used Flip video cameras to collect audiovisual data from each table at which a small group was working. I interviewed participating teachers after each of the lessons and audio recorded the interviews. I kept analytical memos during the course of the research and during the process of data analysis. I transcribed and coded video and audio data.

Communication, communicative practice, and communicative interaction are understood in this thesis as entailing certain shared assumptions about equitable conditions for participation in exchange of utterances and speech acts. The analysis in this thesis led to the development of a theoretical framework for microanalysis of episodes of utterances based in Habermas' (1985a, 1985b) Theory of Communicative Action. This framework, and the associated codes and model are used to generate insights into the functioning of small group interactions. Iterative analysis of transcripts and video data of small group interactions, through the lens of the model and codes, reveals important implications for teachers, students and the wider mathematics education community. These implications include Working Hypotheses about: the technical features of communicative practice in small group interactions; norms and conditions that facilitate and impede participation; how communicative interaction can breakdown; and how to support communication in a classroom setting.

1.1.1 My experience

My experience of teaching mathematics in urban high schools in the United States included teaching the Interactive Mathematics Program ("IMP"), a program designed in partnership with the National Science Foundation ("NSF"). The curriculum was concept-oriented, with open-ended problems, and a focus on constructing meaning through practice. The curriculum's design along with my reform-oriented practice (influenced by complex instruction pedagogy[1] which I had been exposed to during my training in the Stanford Teacher Education Program), provided structure and context for students as they had conversations about concepts, strategies, and procedures. I often noticed that the students asked reflective questions in the process of problem-solving and that they worked collaboratively to develop solutions, test them and justify them to their peers. Of course this did not always happen, and often my classroom felt somewhat chaotic as twenty to thirty students talked and interacted during the regular periods of groupwork. I gained a rich insight into students' thinking and understanding from paying close attention to what they were saying and doing when they worked together. I did this based on my training as a teacher and my studies in philosophy and mathematics and also out of a sense of humility in the face of so many thinking, energetic young people and the challenge of engaging with them rigorously through mathematics teaching.

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[1] http://cgi.stanford.edu/group/pci/cgi-bin/site.cgi

As students worked in groups, my interventions were often focused on modelling reflective, probing and clarifying questions, and supporting students' collaboration. I often reflected on this as a kind of Socratic questioning, conceived of as a dialectical practice focused on fostering independent thought around discursive questioning. I tried to model this type of Socratic questioning for students and encouraged them to ask probing and clarifying questions. The curriculum and teaching approach offered many opportunities to focus lessons explicitly on solving rich mathematics problems (Piggott 2010). I often observed students focused on trying to understand the constraints of problems and trying to develop strategies to solve them.

Teaching in this highly discursive problem-solving environment was engaging and stimulating for me as a teacher. I suspect it was also engaging for many of my students, though I also suspect some of them would express less charitable views about my teaching. These dynamic classroom experiences were often punctuated by the recurrent presentations that students made. Students defended their solutions in front of the class, as other students in the class were encouraged to ask for clarification and justification of the solutions, strategies and explanations. When students were asked to justify their reasoning in the complex solutions they had found, they were often able to do this to good effect. When this kind of teaching was happening in my classroom it excited and invigorated me in large part because so many students seemed engaged and seemed able to develop and display these analytic skills. These kinds of interactions around learning and teaching are a large part of what kept me in teaching despite various professional challenges and personal concerns about problematic ethical and political issues about the institutions in which I was working.

One thing that stood out for me as I taught was the complexity and richness of student interactions around problem-solving. The process of understanding mathematics through these small group interactions seemed to me to be characterised by dialectical processes of questioning, conjecturing and justifying. Observing these kinds of practices seemed to allow me access to the thinking and learning of my students, and to allow for the critical intervention in and development of mathematical understanding through conversation. These reflections and the ideas I had been exposed to in my graduate studies at Stanford, including ideas about the importance of equitable teaching approaches in mathematics and techniques for designing effective groupwork (Boaler & Greeno 2000; Boaler 1998; Cohen 1994) led me to believe that there was a wealth of subtle and complicated social activity that was in need of further study in mathematics education. It was with these thoughts in mind that I set out to engage in research in mathematics education.

Yet, while some of my best interactions in teaching were characterised by engaging students in a rigorous and academic fashion through conversation around mathematical problems and tasks, there were other more troubling aspects of my work. The first of these, obvious to me from the very beginning of my teaching, was around the role I was expected to play as a representative of institutional authority. I was responsible for 'behaviour management' of the students in my classes, as well as responsible for contributing to upholding the normative expectations of the school in all school contexts. At best this involved engaging students in conversations around expectations and consequences. However, there were times when I had to deal with very confrontational situations as well, and I often found this unpleasant and stressful.

Beyond this unpleasantness, I was concerned about my role as a figure of authority for two reasons: First, I was concerned about the ethical justification for contributing to institutional systems of control. This argument seemed largely taken for granted amongst my colleagues, but when it came up it was often framed in broadly utilitarian themes: "educational attainment leads to better economic and social opportunities and is thus an end which can be considered 'good' and so what we are doing in schools is trying to do the most good for the greatest number of our students..." Education for all and college[2] preparation of all students was the accepted ideological rhetoric of the institutional communities I worked in. The actuality was somewhat different. When the actuality became more complex than the ideology allowed, action based in utilitarian ideas (of greatest good for the greatest number) came to the fore. I found this troubling as it was not clear that the means (systems of institutional authority) being employed to achieve the end (college readiness and further education) were ethical in themselves and I was concerned with the potential harm being done to students through psychological coercion and institutional control.

The other reason I was concerned about this role as a figure of institutional authority had more to do with mathematics and the structure of knowledge. I believed that it was important that students learn about the 'authority of the discipline' of mathematics, and I

[2] In the post-secondary sense used in the US

did not want my role as a disciplinarian to impact on their understanding of the authority of the discipline underpinning mathematical knowledge, that is to say, reason and rationality. It seemed to me that there was a real potential for conflation between a hierarchical institutional authority, and the nested, interconnected, and often hierarchically conceived relationships in mathematics. I was particularly concerned that the answers to "Why is this true?" type questions about mathematics were dealt with by recourse to justification and clarification within the discourse and the rationality of mathematics.

These troubling aspects of my work became suddenly more pressing on the morning of 11th October 2002, when a breakdown in normal institutional functioning led to violence in the school in which I was working. This incident and its relation to issues of power and the use of power in schools has since focused my concerns about understanding further the critical potential of educating in schools. On 11th October 2002 at 8:38 a.m. over 90 armed police officers responded to a fight between students on the grounds of the school at which I was teaching. The fight involved several students and was focused around a financial dispute between two of the students. During the police action excessive violent force was used against students who were arrested, thrown on the ground, hit with billy clubs and even had firearms drawn on them by the police. At one point during the incident the police formed a riot line outside the school and advanced on hundreds of students who had been released from class and instructed to go home. Over a dozen students were detained as well as one teacher who had attempted to film the events (SFOCC 2004).

These events did not happen in isolation and could be seen as a more or less direct consequence of political conflict between the school polity and the larger educational bureaucracy about how to achieve the educational missions of the school and the district. The sudden breakdown in the normal running of the school as an institution and the recourse to state-sponsored violence against children to maintain order put into stark and unforgiving context questions about the ethics of the use of power in institutional education. More recently I have thought of this incident as highlighting a fundamental question about schools as institutions of social control in modernity and whether or not they can in fact have an emancipatory capacity. This case deserves in-depth analysis that is beyond the scope of this thesis. I was shocked by the violence and by what I saw as deeply unethical institutional incompetence. While the community was traumatised by the events, in the end we worked together as a faculty and a community to re-establish order and struggled to persevere with the emancipatory mission of the school.

Yet the serious questions raised for me about power and education influenced reflection on my own practice: Was there an emancipatory potential to teaching mathematics, and if so, in what sense? Was my teaching of mathematics related to the issues of power and control embodied in the institutions in which I worked? These questions became even more important in light of the incident described above, which was for me a revelation about the problematic nature of the power exercised by the institutions in which I worked. How did my intention to teach mathematics for social justice correspond to the problematic ways in which power was embodied in the institutions in which I was working?

The tension between wanting to understand dynamic and intricate interactions in the teaching of mathematics on the one hand and the issues of power, control, and emancipation on the other forms an important part of the background that I bring to the research that is the subject of this thesis. This tension serves as an impetus for the questions I pursued and the approaches I employed as well as influencing my analytical perspective. I would like to believe that teaching can have inherent emancipatory potential, but in my experience this is not always clearly the case. This thesis seeks to address understanding interactions in small groupwork teaching in mathematics and whether such an understanding of interactions in mathematics teaching can reveal a critical potential.

1.1.2 Complex instruction in secondary mathematics in England

Complex Instruction is a pedagogical approach that combines a number of interlocking features into a system for designing and managing groupwork in an equitable and effective manner. It begins with a curriculum focused around open-ended mathematical tasks central to the discipline (in this case mathematics). Instructional strategies that foster collaborative groupwork and problem-solving are central to the practice including the deliberate establishment of norms and skills important for working in a group. Finally, it is premised on a re-conceptualization of competency so that all students have a chance to contribute meaningfully and thus gain the skills, knowledge, and confidence that will help them, to explore the rich domain that is mathematics (Boaler 2006; Boaler 2008). This approach, originally developed from a more general educational point of view by Elizabeth Cohen and others at the Center for Complex Instruction at Stanford University in the 1980's and 1990's, has a substantial basis of research and literature exploring the approach conceptually and in its practical application (Cohen 1994; Cohen & Lotan 1997; Staples 2008).

Recently, in England the Complex Instruction approach in mathematics education has begun to be adopted after practitioners were exposed to the research done by Professor Boaler when she disseminated research findings during her time at the University of Sussex (2007 to 2010). This dissemination included a workshop, featuring skilled practitioners from California, which was held at the University of Sussex, leading to the adoption of the Complex Instruction approach by a number of UK schools. Further, the popular NRICH mathematics education website (NRICH.maths.org) highlighted Complex Instruction as one of their monthly themes (May 2010)[3]. Many resources and articles relating to complex instruction in mathematics can be found there as well as a report detailing research on the first steps that schools have taken in adopting the practices. I was privileged to be involved in some of this research and dissemination during my time as a graduate student at the University of Sussex and I located my doctoral research in the context of the preliminary stages of research that grew out of the workshop at Sussex, which led eventually to the Raising Expectations and Attainment Levels for all Mathematics Students (REALMS) research project funded by the Esmée Fairbairn Foundation.

1.1.3 Crisis in mathematics education?

There is an ongoing narrative in the mathematics education literature and in the popular media regarding the inadequacy of current practice, and the need for change in the teaching and learning of mathematics. It is not clear to what extent this crisis actually exists, what its nature might be, or why mathematics education is being framed in this manner. There are other narratives that tend to be more confined to academic literature and the margins of public discourse, which conceive crises in mathematics education differently. What seems to be clear is that all of these narratives serve as an impetus for research to focus on creating change in mathematics education. In this section I briefly sketch elements of these narratives and indicate how this research might be located within the multifaceted narrative of a need for change in mathematics education.

Here I will address two broad categories found in the narratives of the need for change in mathematics education. The first is characterised by policy-oriented narratives of crisis in

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[3] http://nrich.maths.org/7011&part=

mathematics education, and the second by more critical and/or socio-cultural perspective. The first category is conceived as a technical bureaucratic point of view representing what might be called a tacit ideology of the status quo, and the other is from more critical and socio-cultural points of view[4]. I then make the argument that addressing the technical aspects of understanding in small group interactions may serve to uncover a critical potential that can inform a normative basis for change.

From a technical-policy point of view there is an argument for the need to improve teaching and learning mathematics in the two-fold nature of the failure of mathematics instruction within both the UK and the US: 1) Mathematics instruction successfully reaches[5] very few students, despite being regarded as one of the pillars of western knowledge and an essential part of the traditional curriculum; and 2) there is widespread and entrenched acceptance of negative views of mathematics with regards to personal ability and disciplinary difficulty (National Research Council et al. 1989; Goldberg & Harvey 1983; Cockcroft 1982; Ball 2003; Smith 2004).

In a world that is increasingly focused around and dependant upon knowledge production, knowledge management, communications and the corresponding technology, inefficient and detrimental practice within the educational infrastructure with regards to teaching and learning mathematics is a recipe for economic and cultural disaster (from this perspective). To believe that we can do no better in the teaching of mathematics is to accept limits on the potential growth of the economy and to undermine the progressive potential of the project of modernity in all arenas economic and social (Leitch et al. 2007; Hanushek & Woessmann 2007). From this position it makes sense to consider studies of mathematics education which show that certain approaches to pedagogy seem to make possible a more effective alternative to traditional rote mathematics education (Boaler 2002; Schoenfeld 2002). It is an alternative that drives to the heart of the crisis (as it exists from this perspective) currently facing mathematics education in the United States and the United Kingdom.

Recent changes to the National Curriculum in England (QCA Curriculum Division 2008) have brought a renewed focus on aspects of content-related process skills in education

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[4] I am personally more inclined to the more critical point of view, but I think that the policy perspective deserves a certain amount of recognition and respect.

[5] In the sense attainment and participation: low attainment and low voluntary pursuit of the subject

rather than the mere acquisition of facts and procedural skills. This change in emphasis is seen both in the explicit process skills contained in the revised curriculum and in the idea of 'Functional Mathematics'. The process skills in the national curriculum include the categories of Representing, Analyzing, Interpreting and Evaluating, and Communicating and Reflecting. These process skills are integrated into the Programme of Study alongside content and concepts. Functional Mathematics is presented as a subset of the process skills in mathematics that are not necessarily tied to specific content goals. The stated goal of Functional Mathematics is that "learners…be able to use mathematics in ways that make them effective and involved as citizens, able to operate confidently in life and to work in a wide range of contexts"(QCA Curriculum Division 2009). These emphases are potentially complementary to non-traditional forms of teaching mathematics associated with recent reform efforts in mathematics education.

Other more recent changes include: the ending of the National Strategies, which have been criticised in the past for potentially being used to stifle innovative teacher practice with over-regimentation; and the National Curriculum Review, which explicitly states that it is focused on freeing teachers to use their professional judgment (Department for Education 2011). Further, the recent government Task Force report on the state of mathematics education is framed by this technical-political rhetoric of crisis (very succinctly in Michael Gove's introductory comments), and indicates conditional openness to strategies focused on increasing the participation and attainment of all students (Vorderman 2011). These ongoing policy debates, though far from presenting a unified point of view, demonstrate the importance placed on the subject of mathematics education and the widespread acknowledgement of a need to implement ideas from mathematics education research to improve practice and outcomes in teaching and learning mathematics.

In the US there is also 'reform-oriented' work being done on a number of fronts, particularly on the part of the on-going work of the National Council of Teachers of Mathematics (NCTM) and the National Science Foundation (NSF) funded research and curriculum development (NCTM 2000; Arbaugh et al. 2010), as well as a larger policy debate about the need for national standards entailed in part in the Common Core State Standards Initiative[6]. Yet, despite some positive developments, in both the UK and the US mathematics education reform is faced with serious obstacles. The predominant form of mathematics education remains rote learning in traditional forms of pedagogy, and the

[6] http://www.corestandards.org/

crisis remains relatively unchanged from this technical policy orientation (TIMMS 2007; Ofsted 2012).

The second category of narratives addressing potential crises involving mathematics education is couched in the wider issues of social justice in mathematics education and informed by socio-cultural aspects of such potential crises. These critical and socio-cultural perspectives are characterised by viewing the potential crises in mathematics education as complex societal constellations, including (but not limited to) issues of class, gender, race, emancipation and oppression. While some of these issues are in play in the policy-oriented perspective, the general tenor of that narrative either takes for granted, or ignores, certain aspects of narratives of crisis which may in fact be central to the question of how and why mathematics education should or could change. I will attempt to outline a brief sketch of an argument indicating two aspects of narratives of crisis in mathematical education from these alternative perspectives. The first aspect I will address is perhaps the less socio-cultural: the developing and unstable position of mathematical knowledge itself.

The discipline of Mathematics has been engaged in an ongoing struggle to secure its 'foundations' and make clear the nature of the knowledge it professes to have access to. While some may argue that this is merely a philosophical matter, of little import to the day-to-day instruction of mathematics, I suggest that the foundations of mathematics are fundamental to understanding the nature of the social condition of mathematical knowledge. Consider that there are narratives about the nature of mathematical knowledge ranging from realism, Platonism, logicisim, formalism (or conventionalism), and empiricism, to - more recently - constructivism, social constructivism, embodied minds theory, and on and on. This diversity of thought does not necessarily indicate a crisis: a good debate about the nature of mathematical knowledge is surely a sign of a vibrant discipline, critical of its foundations and rigorous in its search for and production of knowledge and truth.

Yet this very vibrancy can be lost (or at least limited to a very small community) in the technical reproduction of knowledge within society unless steps are taken to preserve it. This loss occurs at multiple levels, but two in particular are of interest for the current argument: at the policy level, where the organization of mathematics education in society reinforces the idea of mathematics as discrete and unproblematic knowledge; and at the level of classroom practice where distortion can take the form of teachers viewing mathematical knowledge uncritically and perpetuating the idea that mathematics is merely a set of algorithmic techniques, involving discreet objects, and little more. I am hardly trying to blame teachers in this instance, rather I am suggesting that there is a cultural and systematic nature to what some have described as the conservative cycle (though in slightly different context, referring to the conservatism of practice (Ball 1988)) such that uncritical perspectives on mathematical knowledge are passed from one generation to the next.

The second aspect of the culturally oriented narratives of crises in mathematics education are perhaps slightly less technical in the sense of pertaining to philosophy of mathematics and the implications for teaching and learning, and more about how mathematics education interacts and integrates into the wider social conditions of contemporary western and global civilization. The ideas that form this argument are drawn primarily from Ole Skovsmose, who is considered by some to be the 'father' of Critical Mathematics Education (Ernest 2010). In his book 'Travelling through Mathematics Education' Skovsmose (2005) outlines the nature of the global information economy, pertinent features of globalization, global ghetto-ization, and the role that mathematics education plays as a gate-keeper to the resources and power of the global economy. This point is being taken up by others on a number of different levels, and there is burgeoning research that suggests that the relationship between the social problems of modernity, contemporary globalization, and mathematics education is neither harmless nor unproblematic.

For instance, de Mattos and Batarce (de Mattos & Batarce 2010) make an argument indicting the entire history of mathematics education (in the post-war era) as complicit in the neo-imperialism of the US and the oppressive features of globalization. In this argument they build on Baudrillard's critique of Marx's conception of use value, suggesting that the use value of mathematical knowledge is not more material than its exchange value and that in fact this use value serves as an alibi to justify the consumption of mathematics education. Ancillary to this they make the argument that democracy[7], rather than being corrupted by capitalism, is a "…key link in the creation, development and establishment of capitalism"(ibid. p2). Invoking Baudrillard's theory of sign value they suggest that consumption of mathematical knowledge, under the alibi of use value, is in fact a capitalistic process such that "…economic exchange value (money) is converted into sign value (prestige, etc.)"(ibid., p3). Upon this foundation they build a critique of 'education for

[7] At least the particular form of democracy extant in the later half of the 20th century and its direct historical antecedents.

all' as being an aspect of the increasing dominance of the English language in the post war period, the spread of American[8] style capitalism, and associated ideas of democracy, such that education (and mathematics education in particular) can be seen from within the logic of the commodity as complicit in the inequity inherent in contemporary global society. These arguments fly in the face of the (perhaps naïve?) beliefs of many practitioners who are committed to the betterment of society through education and raise serious and alarming questions as to how and why these oppressive features of mathematics education may exist, and if and how they may be mitigated.

The work in my doctoral thesis is focused on examining and understanding small-group interactions in the context of new forms of teaching that are intended to allow more students to have success in mathematics education. While the analysis will focus on technical aspects of interactive communication, it will also be developed using a Habermasian theoretical framework that seeks to allow for the relation of the communicative action of developing understanding in small groups to the social features of particular classroom practices. The theories, developed in this manner, are amenable to being used to relate classroom practice to the systematic features of educational institutions and to wider societal issues (although this is beyond the scope of this thesis). In this way the research is focused on producing theories and models that may facilitate an understanding of the intersection of the technical practice of teaching for mathematical understanding (using groupwork) with the complex social issues at play in the development of students as learners of mathematics and the development of mathematical communities in a wider social context.

1.2 The focus of this research

The focus of this research is examination of the small group interactions of students in mixed ability classrooms in year seven mathematics classes, with the intent to develop ways to understand these interactions and why/how such understandings may be important. While the thesis grew to be focused on the use of Habermasian social theory, both at the level of theory development and also methodologically, I originally began by approaching this question from a perspective of situated theory (Greeno 1998). It is my position that these theoretical positions can be used in a complementary fashion in order to deepen and

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[8] American here refers to the United States of America

broaden insights from the research. This issue of complementarity is addressed in Chapter 2 Section 2.4.

Situated theory suggests that one cannot look at the cognition of subjects in isolation if one wants to understand the form of thought and its relationship to action. Rather, one should look at the cognition of subjects in relation to the 'situated' social and contextual aspects which frame, influence and constitute cognition. Situated theory suggests that there is an ecology of learning, and that important features of such an ecology can be thought of as constraints and affordances (Greeno 1998; Lave & Wenger 1991). The use of Complex Instruction can be seen as an explicit attempt to influence the constraints and affordances of the classroom environment. The aim of the explicit expectations contained within the norms and roles communicated by complex instruction is to create opportunities for productive groupwork. However, from a critical mathematics education perspective, the use of complex instruction by the teachers in this research can be seen as an Imagined Situation, or ideal goal, while what is actually taking place in the classroom is the Arranged Situation, or the actuality of practices arranged in the tension between the Current Situation and the Imagined Situation (Skovsmose 1994). The aim of this study began as an attempt to examine the interactions that are actually taking place (the arranged situation) in the small groups focused on problem-solving and to analyze the evidence of norms and expectations influencing or acting as constraints and/or affordances, and to examine how and to what extent these features relate to productive participation in the learning of mathematical knowledge and practices. The consideration of the social at the heart of this framework allows for the consideration of not only social features at the level of the site, but was also intended to allow the consideration of how these factors might relate to wider social issues such as the technical and cultural crises of mathematics education addressed in the previous section.

1.2.1 The elevator pitch or the hair dresser question

In an attempt to clarify what it is I am trying to do, I considered how I would explain it to someone unfamiliar with mathematics education research or the social sciences. Such an encounter might go something like this:

“Lay-person interlocutor: So, what is it that you do?

Author: I'm a researcher in Mathematics Education, currently finishing my Doctoral Studies, thanks for asking.

Lay-person interlocutor: Oh? That's interesting- and what is it that you are researching?

Author: Well, I am trying to understand how fairer approaches to teaching mathematics relate to technical features of students learning from a communicative perspective.

Lay-person interlocutor: That's interesting, I was never any good at mathematics in school.

Author: Well, I guess that's sort of part of the problem, right? Mathematics is an important part of our society and daily lives, and yet most people say they feel they're no good at it, right? So I reckon it has to do with how it's taught and how students learn. So I'm looking at how students learn in mixed ability groupwork, trying to understand how they interact.

Lay-person interlocutor: Huh, that's actually kind of interesting.

Author: Success! "

1.3 The research questions

The focus of this study, outlined above in Section 1.2, is the driving force behind the research questions, which guide the choice of research instruments, the collection of data, and the analysis of findings. It should be noted that the focus of the research changed (slightly) in its implementation and that this is reflected in the sub-questions. This change in focus happened for a couple of reasons. First I realised that I did not want to focus on ground that had already been well covered (albeit in slightly different contexts), and second I became increasingly aware that my analysis was trying to locate a critical relationship between technical understanding of small group interactions and wider social issues particularly regarding social justice.

The original question and first sub-question posed here are premised on the idea that as non-traditional practices are adopted students and teachers encounter a changing set of norms and patterns of interaction. The change in practice leads to changing constraints and affordances available to the teacher and students in the mathematics class (Greeno 1998).

The aim here was originally to examine how the participants, and the students in particular, navigate these changing pedagogical factors.

Main Question: How can we understand student interactions in the context of small group problem-solving in mixed ability year 7 mathematics classes adopting elements of Complex Instruction?

This question, conceived from a perspective of situated theory, was trying to flesh out the ideas of practices, constraints and affordances, as well as examine the details of the interactions that constituted the learning and teaching that are taking place in the classroom. This question was supplemented by a sub-question to help focus the research without losing the broad aims of the primary question.

Sub-question: Are there patterns of interaction, both among students in small groups and between students and the teachers? If so, what are their characteristics?

These questions were also premised on my experience of teaching in rich discursive environments. There was an important and intricate give and take of ideas couched in the discursive practice I participated in, and in which my students participated, when I was teaching mathematics in the US. I wanted to see if similar features in English Schools could be analyzed and understood in a way that made sense of the complex interplay of language, mathematical knowledge and practices, and issues of social justice. The focus on communicative action came later in the process of data collection and analysis, but this research began with these questions from this perspective. At a later stage in the analysis, I added a further question:

Question: What is the critical potential of understanding student interactions from a perspective of communicative action?

This later focus reflects an acknowledgement of my preoccupation with issues of social justice, critical theory and critical mathematics education (as already noted in this chapter). It also reflects what I consider to be the crucial feature of the ways of understanding developed in this work: understanding from a communicative perspective can act as a basis for critique.

One of the aims of the microanalysis in this thesis was to gain insight into how practices on the level of small group interactions constitute and shape the not only particular knowledge and practices on the local level, but also hopefully to gain insight in a manner which may inform analysis of wider macro-level issues of patterns of production and reproduction of mathematical knowledge and practices. In particular, addressing ongoing issues of inequitable attainment and participation in England (Boaler, Altendorf, Kent 2011), as well as wider issues of social justice and mathematics education, was felt to require an understanding which was both technical and also developed in a manner which points towards analytical links between macro and micro features of the social reality of mathematical learning and teaching in schools. The extent to which this was realised is taken up again in considerations of future work based on the research and findings of this thesis in Chapters 9 and 10.

In the following chapters I will review relevant literature from mathematics education and the theory of communicative action (Chapter 2), discuss methods and methodology including an integrated analytical strategy (Chapter 3), situate the analysis of small group interactions within the wider classroom context (Chapter 4), discuss the analysis of small group interactions from open-coding and constant comparison through the development of analytical statements and hypotheses (Chapters 5 through 9), and finally discuss the findings and implications of this study (Chapter 10).

Chapter 2: Literature review

In this chapter I consider key features of the Theory of Communicative Action (hereafter TCA) for the analysis of evidence of intersubjectivity and their relation to established theoretical perspectives in mathematics education in order to argue that the theory of communicative action can be used in conjunction with, at least, situated perspectives, and also with other theories of communication and interaction in mathematics education. I argue that concepts of intersubjectivity based in Habermas' Theory of Communicative Action can be coherently used alongside insights from other theoretical approaches in the analysis of small group interactions in the context of year seven mathematics classrooms in the UK adopting aspects of complex instruction. This is important because I seek to use insights and theoretical tools from research conducted in situated theory, and also to construct theories that allow for the possibility of complementarity to interactionist, situated and socio-cultural perspectives in mathematics education.

There are two sections to this literature review - one focused primarily on issues of communication, intersubjectivity and mathematics education, and the other on changing teacher practice and 'reform-oriented' mathematics teaching. The first is in four parts and begins with a discussion of communicative action. I address the ideas of speech act theory and Habermas' and Apel's conceptualization of linguistically constituted knowledge in the context of communicative intersubjectivity wherein understanding of meaning is achieved through a process of consensus which is never fully realised. This is used as the context for a closer look at how intersubjectivity expresses itself and a discussion follows of the nature of intersubjectivity from a Habermasian viewpoint (in the context of the Theory of Communicative Action (Habermas 1985a; 1985b)) and the associated potential themes for analysis. This leads into a discussion of the nature of intersubjectivity in the literature of mathematics education followed by a discussion of how to use the Theory of Communicative Action in relation to other theories in mathematics education. Other theories addressing theoretically and empirically related concepts in mathematics education are discussed. They include: a discussion of Alro and Skovsmose's work on dialogue from a position of critical mathematics education; further discussion of interactionist and socio-cultural theories; and finally a look at situated theories that will allow for the possibility of positioning of the use of concepts of communicative action in the analysis of issues of intersubjectivity in relation to the major theoretical points of view that address similar issues in the field of mathematics education.

The second section of the literature review addresses other concepts and literature considered important for an understanding of the issues at play in the research context beginning with a discussion of the notion of reform oriented mathematics practices (a concept rooted in US policy developments over the past two decades), and including a discussion of research in groupwork, problem-solving, complex instruction, and the Railside research. Theories of teacher learning and practice are discussed to help frame some contextual issues specific to the research in this thesis. Finally, some sociological theories that address the relationship between microanalytic features and macro sociological analysis are discussed in the context of the decision to use the theory of communicative action in this study. In conclusion, an argument is made for the use of concepts of intersubjectivity, based in theories of communicative action, in the analysis of small group interactions in the context of year seven mathematics classrooms in the UK adopting aspects of complex instruction.

2.1 The Theory of Communicative Action: Habermas and Apel

Communicative action, as articulated by Habermas in the Theory of Communicative Action (TCA), is one of four concepts addressing the rationality of an agent's action. While this study is primarily focused on using concepts of communicative action (and its emphasis on the necessity of assumptions of equity for understanding meaning) to understand student interactions, it is important to understand Habermas' distinction. The first three outlined are teleological or strategic action, wherein an actor is focused on achieving certain ends by deciding amongst courses of action that act as means to that desired end. The difference between teleological and strategic is that the means to achieving the desired end in strategic action include at least one other goal-directed actor. The essentially means-end rationality this position entails is guided by maxims of utility or expected utility. The second concept of action is normatively regulated action, wherein members of a social group orient themselves and their actions to essentially common values. This model of action entails expectations on the part of group members that they are entitled to expect certain behaviour from other members of the group. The third model of action is dramaturgical action and is essentially an expressive model of action wherein the actor is presenting themselves to an audience. This model of action is concerned mainly with the stylised regulation of access to one's own subjectivities. In contrast to these three models of action, Habermas presents a fourth, communicative action, which he asserts is distinct in that where each of the three previous models refers to either solely the objective world (strategic), primarily the subjective world (dramaturgical), or to the objective and the social worlds (normatively regulated), communicative action is said to refer to all three of these worlds (the objective, the social, and the subjective) simultaneously.

> Finally, the concept of communicative action refers to the interaction of at least two subjects capable of speech and action who establish interpersonal relations (whether by verbal or by extra-verbal means). The actors seek to reach an understanding about the action situation and their plans of action in order to coordinate their actions by way of agreement. The central concept of *interpretation* refers in the first instance to negotiating definitions of the situation which admit of consensus. As we shall see, language is given a prominent role in this model. (Habermas 1985a)

As indicated by the above quotation, language is essential in the model of communicative action. A linguistic medium 'that reflects the actor-world relations as such' is taken as a presupposed condition for the functioning of the model. Achieving understanding is taken as a mechanism for coordinating action. This is done by recourse to the concept of validity claims. Habermas asserts that in communicative action, utterances (or speech acts) make three simultaneous, if not necessarily explicit, validity claims; that the utterance is objectively true, that it is normatively right, and that the speaker is subjectively being sincere. In this manner the participants can negotiate the meaning of a particular action situation and reach interpretative agreement. If another party in the interaction presents a definition of the action situation as distinct from the one held by the actor, or agreed upon by the group engaged in communication, further negotiation is required so that the various interpretations of the situation can be brought 'sufficiently into alignment'. The necessity of this relates to one of the preconditions for communication entailed in what is referred to as the ideal speech situation, namely that no one in communication has a monopoly on correct interpretation (ibid. p100).

This ideal speech situation is less emphasised in Habermas' later work on the Theory of Communicative Action, but is more explicit in his early work on the pragmatics of social interaction (Habermas 2002). In this formulation the ideal speech situation consists of three aspects: namely 1) that all linguistically capable participants are allowed to take part in discourse; 2) that there is a symmetrical opportunity for participants to question any assertion; 3) that there is a symmetrical opportunity for participants to introduce any assertion into the discourse; 4) that all participants are allowed to express their attitudes, needs and desires; and 5) that none of the previous conditions are affected by the use of coercion (power). It should be noted that this ideal speech situation refers to discourse, which for Habermas is a particular aspect of communication that occurs when the (mostly tacit) consensus at the heart of communicative action breaks down. This breakdown occurs when participants fail to agree on the validity claims associated with the utterances that constitute the interaction. When this happens, communication stops aiming at the exchange of information and interpretations of the action situation, and becomes focused on re-establishing the consensus, which consists of bringing the various interpretations sufficiently into alignment that the actors may continue to collaborate around shared goals. Habermas has been criticised for the idealist nature of the ideal speech situation, but it should be noted that this notion is explicitly conceptualised as a counter-factual norm which is never actually realised but which acts as a necessary presupposition for the consensus theory of truth. The counter-factual nature of these conditions is implicated in the fallible and fragile nature of communication:

> Stability and absence of ambiguity are rather the exception in the communicative practice of everyday life. A more realistic picture is that drawn by the ethnomethodologist - of a diffuse, fragile, continuously revised and only momentarily successful communication in which participants rely on problematic and unclarified presuppositions and feel their way from one occasional commonality to the next. (Habermas 1985b)

Karl Otto Apel, writing in the same tradition as Habermas, takes the ideas of communicative action further with regard to scientific knowledge, whereas Habermas concentrates primarily on ethical and political issues (Delanty 2005). This is important if one is to use communicative action as a way to understand the linguistic practices of mathematicians and mathematics learners, as the consideration of the nature of scientific knowledge is closer thematically to the understanding of mathematical knowledge than Habermas' main themes of social and ethical knowledge. In particular Apel addresses the role of consensus as a counter-factual norm inherent in the structure of the language of which knowledge is constituted (ibid.). This concept of consensus as a counterfactual norm is central to the coherence of the use of TCA to analyze small group-interactions in that it is a precondition which abductively allows sense to be made out of the fluid formation and disintegration of consensus. That is to say that the theory of communicative action can be useful in the examination of student interactions on the level of their utterances, but that for the claims of the theory to be coherent such a norm must be presupposed, though its ideal nature means that it does not exist as such empirically except in a momentary and fragile manner.

2.1.1 Intersubjectivity and communicative action

The relation between intersubjectivity and the theory of communicative action is that linguistic intersubjectivity is the medium that is at the core of communicative action. Communicative action consists of several parts, the coordination and negotiation of the action situation, the coordination of goals and means to attain goals between actors (in a non-strategic manner), the parallel pursuit of these means and ends, and discourse wherein the consensus that underlies the coordination of interpretations, goals, means and ends can be re-established should it become problematic in the course of communication. Consider the quote from Dews' *Logics of Disintegration: Post-structuralist Thought and the Claims of Critical Theory*:

> For Habermas, as we have seen, linguistic intersubjectivity is the medium in which claims to truth, in the cognitive sense, as well as claims to rightness and authenticity can be raised and arbitrated. (Dews 1987)

Intersubjectivity exists within the processes of communicative action for Habermas. In the processes of communication towards reaching understanding and discourse towards establishing and re-establishing the consensual preconditions for understanding, the interplay of linguistic utterances of actors constitutes intersubjectivity from the points of view of Habermas and Apel (Habermas 1985a, 1985b; Delanty 2005).

2.1.2 Communicative action, communication, and discourse

Communicative Action is the coordination of action by multiple goal-oriented actors through a process of cooperative interpretation. This is to say that action is coordinated to achieve goals through the achievement of intersubjective understanding (Habermas 1985a). This concept is the focus of Habermas' work and rests on his rejection of the mind body dualism of the philosophy of consciousness. For Habermas meaning and understanding are inextricably linked such that understanding the meaning of an utterance implies: "1) The recognition of its literal meaning; 2) The assessment of the speakers intentions; 3) Knowledge of the reasons which could be adduced to justify the utterance and its content and; 4) Acceptance of those reasons and hence the utterance" (Finlayson 2005). Thus communicative action, defined as coordinating action through reaching understanding, is characterised by interactions that have the features of Habermas' theory of pragmatic meaning, including multiple tacit validity claims referring simultaneously to objective, subjective and social realms. Communicative action, like other models of action, has as its purpose the achievement of the goals of the actors involved. However, in communicative action this is achieved through understanding (as defined by Habermas' theory of pragmatic meaning). This is the key insight for realizing the usefulness of communicative action as a theoretical lens for evaluating interactions in the context of complex instruction.

The reason this is key is because of the many aspects of complex instruction that seem to map loosely onto the theory of communicative action. Normative elements of the classroom situation are very explicit and focus the students on working cooperatively to achieve their tasks. They are encouraged to ask questions, listen to each other and provide justification for their ideas. There is an emphasis on consensus and agreement. Further, the tasks themselves in complex instruction are focused on conceptual content and problem-solving so that achieving the task entails to a greater or lesser degree the understanding of conceptual content. In the next section analytical themes based in the Theory of Communicative Action will be reviewed and the theoretical tools for addressing them are discussed. In particular the argument is that development of an intersubjective understanding of classroom based groupwork interactions, based in the TCA, allows for the potential to analyze the relationship between characteristics of small group interactions and issues of classroom practices, and potentially also local institutional features and wider social issues.

2.1.3 Analytical themes based in the Theory of Communicative Action

Examining interactions in small group mathematics problem-solving from a communicative perspective provides a range of potential analytical themes and the theoretical underpinnings for addressing them. In this section I consider the idea of systematically distorted communication and the concept of the 'colonization of the lifeworld' and their potential relationship to analysis of small group interactions. In Chapter

8, examination of breakdowns in communicative action will provide insight into obstacles to learning and teaching using mixed ability small groups. Further, it will create a context for understanding how participants act to establish more productive situations for small group interactions, a topic which is addressed in depth in Chapter 9.

The issue of status and its potentially negative impact on learning has been recognised for decades. A perspective of communicative action in the field of mathematics education has the potential to reveal technical features of the processes of communication that are affected by issues such as status in small groups. By examining the ways in which distorted communication can express itself in classroom activity, one can begin to understand how to promote students' access to communicative rationality, and the development of an epistemic rationality that is the foundation of mathematical knowledge.

2.1.4 Concept: systematically distorted communication

Originally employed as a way to negatively explore a concept of communicative competence, systematic distortion was an idea that Habermas emphasised less as he developed his mature theory of communicative action and the concept of colonization of the lifeworld. However it is still useful to consider in that it explicitly deals with the relationship between ego-identity and communicative action. Consideration of how understanding may be undermined or fail on the level of individual participation in social interactions is a vital perspective with regard to the analysis of school learning, and especially when looking at school learning of mathematics, which is often characterised by low rates of participation and attainment, and widespread and persistent negative personal narratives. In systematically distorted communication, participants employ strategies of communication that prevent the achievement or maintenance of intersubjective understanding.

In his earlier thinking, Habermas approached the idea from a psychoanalytic perspective. He suggested that in some family situations, where there was an uneven distribution of power identity, conflicts could affect internal organization of speech such that conflict is avoided through various strategies of self-deception (Habermas et al. 2002). While this thesis is not examining family situations, the argument for the relevance of this psychoanalytic perspective is that the forms of rationality that are developed in family relations serve as foundational models for rationality in other social settings, including schools. I will briefly discuss a few of the strategies with a view to noting the importance of these ideas in an educational setting generally, and to show how they relate to Habermas' broader ideas of the systematic colonization of the lifeworld.

From this psychoanalytic perspective, as articulated by Habermas (2002), ego-identities are developed and maintained in the context of family relations (primarily - but also in other social arenas). The issue of 'securing one's identity' is addressed by Habermas. Without addressing the details of his argument I want to note two issues. First, when identity management in social groups breaks down (and this can happen in a number of different ways), identity conflict ensues and in a symptomatic situation where these conflicts cannot be worked out discursively, the "pressure of identity conflicts is shifted onto the internal organization of speech where it is stabilised, but remains unresolved" (ibid. p164). Second, there are communicative strategies, such as joking and irony, that may mediate power relations so as to "clear the ground for discourse" (ibid. p162).

Addressing the first in slightly more detail, in an attempt to penetrate its opacity and understand its implication for educational analysis from a communicative perspective, I note that the communicative disturbance resulting is such that instead of being able to reach intersubjective understanding, participants act towards each other in 'thinly veiled strategic fashion' (at least when the pretence of consensual participation is still required). This is an interesting idea, which suggests that a participant may find it necessary to participate in a dysfunctional community in order to secure and maintain their identity. However due to some systematic distortion, the mutual recognition of validity claims that would be characteristic of a properly functioning intersubjective situation may only be possible through the tacit violation of one or more of these validity claims (ibid. p164). An example of this is the establishment of pseudo-consensus such that consensus appears to be maintained but in fact communication is flawed due to the avoidance of addressing validity claims. This can be done in a number of ways, which basically boil down to a participant not giving a legitimate answer when a validity challenge is made. This is all rather abstract, but examples from classroom practice are not hard to imagine: class-clown behaviour in the face of teacher or peer task-focused questions; idle off topic chatter that persists even in the face of repeated attempts by teacher or peers to engage in task-oriented conversation; silence/non-participation; ignoring the interlocutor; etcetera. In fact one could probably develop an entire taxonomy of pseudo-consensus strategies by polling any group of teachers at the pub on a Friday evening. However, I do not want to be too negative. In fact I would suggest that the negativity stems from Habermas' negative strategy for investigating conditions of communication. Further, I would not want to suggest that I am blaming the students, or in fact any of the categories of participants, in particular. After all, Habermas is describing a *systematic* distortion of communication.

These ideas of systematic distortion and the potential for irony and humour to promote discursive space are useful in the analysis of student interactions in this study and feature in the analysis of data in Chapters 8 and 9. In the next section I examine how Habermas accounts for the breakdown of communicative action in his later works.

2.1.5 Colonization of the lifeworld

The Colonization of the lifeworld is a concept developed by Habermas describing the negative effects of a diminishing lifeworld. In this model the lifeworld, which acts as a reservoir of intersubjective meaning through communicative action, becomes dominated by the strategic action of the system. Since according to Habermas' theory the logic of strategic action is based on the logic of communicative action, the colonization of the lifeworld results in effects, which begin at the local level of the lifeworld and then cause the system to stagnate and potentially fail. The symptoms of colonization of the lifeworld are called social pathologies,[9] and include:

1) Decrease in shared meanings and mutual understanding
2) Erosion of social bonds – disintegration
3) Increase in feelings of helplessness and lack of belonging – alienation
4) Consequent unwillingness to take responsibility for their actions and for social phenomena – demoralization
5) Destabilization and break down in social order – social instability
(Finlayson 2005)

These pathologies are related potentially to important understandings of mathematics education (such as learning in the form of shared meaning, mutual understanding, feelings of helplessness, and lack of belonging) and address the issue of identity formation. In this section I will explore these ideas (systematic distortion and colonization of the lifeworld)

[9] The use of the term pathologies harkens back to the psychoanalytical approach of systematic distortion of communication which he developed originally and which is related to the concept of the colonization of the lifeworld.

and use them to analyze the data concerning communication breakdowns gathered for this study. This is part of the overall intention of this thesis to recognise how developing theories for understanding small group interactions can inform potentially related wider social issues.

2.1.6 Lifeworld: Husserl (Phenomenology) and Habermas (Critical Theory)

The lifeworld is a central concept of Habermas' theory of communicative action originally developed by Edmund Husserl.[10] It can be thought of as the cultural norms and practices of a given social group. According to the theory these norms and practices are established and maintained by intersubjective standards. It is the constellation of experiences, and linguistic and cultural frameworks that allow participants to make meaning of their everyday lives. It is here that notions of intentionality come into play as this notion rests on a conception of participants as making meaning in an intersubjective manner (Habermas 1985a). There is a technical notion of empathy used in Husserl that is interesting to note especially in connection with ideas such as "The productive agency that drives collaboration" put forward by Daniel Schwartz (1999). The definition of empathy used by Husserl is that of an ascription of intentionality to other participants in communicative situations. This is recognised by Habermas in his development of the concepts of intersubjectivity, the lifeworld, and communicative action. It is also noted within theories of communication in mathematics education such as those discussed previously, developed by Sfard and Skovsmose and others (Sfard 2008; Alrø & Ole Skovsmose 2002).

In Habermas the lifeworld serves as the unregulated context for communicative action. Thus it exists in a dialectical fashion with communicative action, in that on the one hand it serves as the store of shared understandings that make communication possible and on the other hand it is the product of the intersubjective understanding produced in the process of communication. In this manner the lifeworld serves as a kind of generative store of meaning. This is important because the systematic structures of society depend on the vibrancy of the lifeworld, as do arguably the formal rational systems of scientific and mathematical knowledge. Further, the lifeworld, sustained as it is by effective communicative action, can become stagnant and/or defective through the systematic distortion of communication and the colonization of the lifeworld by the system. While I

[10] It is interesting to note that the original formulation of the lifeworld was as the everyday rationality of normal people in contrast to the theory oriented rationality of scientific practice.

have addressed briefly the concept of systematically distorted communication, the notion of systematic colonization requires some further comment.

What are the implications for mathematics education from these concepts? If, in certain circumstances, I can interpret teaching and learning as the cultivation of the rationality inherent in communicative action in the service of developing mathematical knowledge and practices within society, then the implications of evidence of communicative breakdowns, both locally and systematically, are that such cultivation may fail or proceed in an ineffective manner. An important intersection between situated theory, as articulated by Wenger (1990), and Habermas in the TCA, is the parasitic dependence of the system on the meaning making that occurs within the lifeworld (or within 'communities of practice' in situated theory). The negative implication is that the system's tendency to interfere with the conditions for communicative action in the lifeworld through over-regimentation is essentially undermining the system itself. Two ancillary questions are implicated: 1.) Is the meaning making occurring in mathematics classrooms within the realm of the lifeworld?; and 2.) What is the connection between the everyday rationality of the lifeworld, the rationality of the discipline of mathematics, and the rationality of the system (and in particular the institutional settings in which mathematics education and mathematics reside as disciplines)? These questions are beyond the scope of the current study. However, the conceptual framework of Habermas' TCA may allow the theory developed in this study to be used to address these questions in other research.

2.2 Intersubjectivity and mathematics education

Intersubjectivity is an important issue in mathematics education as it is a sign of a significant shift in the historical conceptualization of the knowing subject and thus also in the nature of knowledge. While a detailed discussion of the changing conceptions of the subject in western thought over the past century is beyond the scope of this review, signs of the change can be seen in the shift of the mathematics education literature from conceptualizations initially based more or less solely in psychological theories towards theories that attempt to re-conceptualise the field using social and socio-cultural ideas (Lerman 2006). This shift has been driven by awareness that there are complex issues at play in mathematics education, which could not be accounted for using a solely psychological perspective. Intersubjectivity becomes a central, if at times ill defined, concept in socially oriented theories that seek to understand the nature of learning and teaching mathematics. It is in seeking a more technical, empirically based, understanding of how intersubjectivity presents itself and how it affects/relates to the constitution of knowledge, that the theory of communicative action may be useful.

Thus the issue may be stated as a question: what is the nature and the role of intersubjectivity in the constitution of knowledge and the knowing subject? Intersubjectivity is a concept that appears in a variety of contexts in the literature of mathematics education. It is recognised as a facet of the educational situation by authors working from constructivist points of view, by interactionists, and by socio-cultural researchers, amongst others. The positioning of researchers with regard to this issue and the surrounding theoretical and methodological arguments has arisen recurrently in the literature. Stephen Lerman for instance raised the issue explicitly in an article that challenged the theorizing of social constructivists (as based too much in radical constructivism and therefore incoherent with the primacy of the social that Lerman relates to concepts of intersubjectivity) and at the same time outlined several features of intersubjectivity in mathematics education (Lerman 1996). While for the moment wishing to avoid becoming embroiled in a complex multi-sided dispute over the relative use of theory, it is interesting to note the three features that Lerman discusses regarding intersubjectivity in mathematics education.

The first is the idea that subjectivity is constituted through social practices and thus can be said to come into existence through intersubjective processes. This idea resonates with ideas from situated theory, which see identity as a process of coming to belong to a community of practice. I shall return to the idea that the concepts of intersubjectivity can be complementary to ideas from situated theory as expressed by Lave and Wenger. However this idea that subjectivity is socially constituted is something which needs to be articulated in more detail. How precisely do intersubjective processes lead to the formation of a more or less autonomous subject? Lerman uses an interpretation of Vygotskyian theories of enculturation, internalization, and the zone of proximal development to address these questions. I would argue that there is still a technical linguistic element that could benefit from analysis of the intersubjective processes from a point of view of the theory of communicative action should the data be able to support such an interpretation.

The second feature that Lerman points out is the notion of cognition as situated in practices. This idea seems to be complementary with Habermas' ideas as well as Dew's interpretation of intersubjectivity from a Habermasian position. Further, within the theoretical literature of mathematics education, Sfard and others (Sfard 2008; Sfard & Kieran 2001) have argued that cognition ought to be thought of as a process of communication. While this claim is to a certain degree contentious as it takes a position which undermines traditional cognitive approaches that see language as a sign of underlying cognitive functions, it is important to note that as a metaphor it is pointing to the strong relationship between the conditions of satisfaction of intentional mental states and the conditions of satisfaction of communicative utterances (Searle 2010).

The final feature that Lerman highlights is the idea of mathematics as cultural knowledge. In this situation Lerman again uses ideas founded in Vygotsky of the pre-existing social structures that are the force behind the development in participants of knowledge that is part of the cultural tradition of mathematics. It is interesting to note that he sees this function as non-deterministic yet having a real force that has a more or less direct impact on the development of knowledge in participants. It should be noted that this is not necessarily a transmission concept but is related to the former idea of cognition as situated in practices.

Lerman makes a coherent argument for the separation of cognitive traditions and socio-cultural traditions, asserting that it is incoherent to assert that a radical constructivism can be a primary foundation from which to understand the functioning of social processes as having force in the development of knowledge and subjectivity. However, not all authors take such a strong position. Bauersfeld (1994) makes a case for interactionist perspectives as a middle way.

Table 1 Bauersfeld's Schema of Perspectives (1994)

Individualistic Perspectives	*Collectivist Perspectives*
Learning is individual change, according to steps of cognitive development and to context. Prototype: Cognitive Psychology.	Learning is enculturation into pre-existing societal structures, supported by mediator means or adequate representations. Prototype: Activity Theory.
Interactionist Perspectives Teacher and students interactively constitute the culture of the classroom, conventions both for subject matter and social regulations emerge, communication lives from negotiation and taken-as-shared meanings. Prototypes: Ethnomethodology, Symbolic Interactionism, Discourse Analysis (Pragmalinguistics).	

As can be seen from the chart above the interactionist perspective as articulated by Bauersfeld is quite close to the theories of communicative action as articulated by Habermas, with their attention to the practices of ethnomethodology and pragmalinguistics. However it should be noted that the literature on interactionist perspectives hardly addresses this overlap with Habermasian interests by explicitly referencing him or the theory of communicative action. Rather there is articulation and reference to speech act theory and pragmatic semantics and other (primarily sociological) traditions that overlap with the interests of elements of Habermas' reconstructive approach to theorizing social science.

2.3 Related and potentially complementary theories in mathematics education

Over the past 20 years approaches characterised by a focus on the socially constructed nature of mathematical knowledge have been developed, implemented and researched in some mathematics classrooms. With this focus has come an emphasis on mathematical discourse, complex problem-solving, and mathematics in context. There have been several notable strands of theoretical work that have contributed to understanding learning and teaching in light of these relatively new foci. Socio-cultural and situated theories have contributed heavily in this field and can be seen as providing a solid foundation for theorizing learning as a social activity (Cobb & Bauersfeld 1995; Lave & Wenger 1991; Kirshner & Whitson 1997; Lerman 1994).

Research into mathematics education has increasingly focused upon communication, discourse and language in the past 20 years (Gutiérrez & Boero 2006). This has happened alongside the shift from purely psychological and cognitive models towards models that incorporate social aspects of learning mathematics. There are a number of important perspectives on communication and its relation to mathematics education to be touched on here as context for the argument that the theory of communicative action has something new to offer in the analysis of complex instruction style mathematics teaching. In this section I address some of Sfard's ideas concerning cognition as communication, and then note some pertinent ideas from Skovsmose and Alro addressing communication from the perspective of Critical Mathematics Education. These ideas should set the stage for the analysis of small group interactions from a standpoint of communicative action and specifically how complex instruction supports this type of communication. Finally there is an examination of how understanding can be seen from this perspective so that a technical connection can be made between issues of equity and learning.

One of the most interesting and perhaps controversial perspectives with regard to communication in mathematics education is that developed by Anna Sfard over the past 10 years. This theoretical framework, which Sfard refers to as "Commognition", is characterised by the position that cognition or thinking can be seen as a special case of communication, specifically communicating with one's self (Kieran et al. 2003; Sfard 2008). One of the entailments of this conceptualization of cognition that Sfard points out is that thought and speech are "…inseparable aspects of basically one and the same phenomena, with none of them being prior to the other" (Kieran et al. 2003). Two further things to note in Sfard's treatment of Commognition are: 1.) The idea of learning as initiation into a discourse; and 2.) Mediating tools and meta-discursive rules as factors giving discourses their distinct identities. The first idea seems familiar from situated theories of learning wherein learning is seen as a process of becoming a member of a community of practice (Lave & Wenger 1991). The second idea is interesting because it sets out to identify specifically what defines a discourse such as mathematics.[11] The idea of mediating tools as part of what defines a discourse requires a view that symbols (including language) are essential to communication and thus cognition. Alongside this and perhaps most apropos with regard to this thesis is Sfard's idea of meta-discursive rules.

> …meta-discursive rules are what guide the general course of communicational activities. It is noteworthy that meta-discursive rules are mostly invisible and act 'from behind the scenes'. Because of their implicit nature, and in spite of their ubiquity, they have not been given any direct attention in the past. (Kieran et al. 2003)

This concept of meta-discursive rules is perhaps the most relevant of the ideas that Sfard has put forward to the concepts being developed in this thesis. If one can interpret the interactions of students engaging in small group problem-solving from a point of view of communicative action, then one may be able to make an argument that this technical understanding of communication can serve as a basis for an understanding of the transition from the everyday discourse of the lifeworld to the specialised discourse of mathematics. The essential element of this argument is that communicative action is a basic model for the meaning that is generated from the lifeworld and upon which the meanings and rationality of systems and institutions rest. If one then accepts that mathematical discourse is a special type of communication with special meta-discursive rules, then one may be able to understand what needs to be done to build upon the rationality of communicative action to create the potential for mathematical discourse in classrooms. Understanding classroom communication in terms of validity claims and conditions of discourse may help to understand the development of mathematical rationality out of everyday rationality.

Analyzing interactions in mathematics classrooms that are using complex instruction from a framework of communicative action can be informed by related ideas from work in critical mathematics education (Skovsmose 1994; Alro & Skovsmose 2004). This is especially the case as the theory of communicative action is (in some ways) an extension of Frankfurt school critical theory and Skovsmose has been at the forefront of developing Critical Mathematics Education (CME) over the past 25 years. CME, according to Skovsmose, is concerned with:

[11] Mathematical discourse is defined by Sfard as "…a special form of communication known as mathematical" (Kieran et al. 2003).

> …how mathematics in general influences our cultural, technological, and political environment, and the functions mathematical competence may serve. For this reason, it not only pays attention to how students most effectively get to know and understand the concepts of say, fraction, function, and exponential growth. Critical Mathematics Education is also concerned with matters such as how the learning of mathematics may support the development of citizenship and how the individual can be empowered through mathematics. (Alro & Skovsmose 2004)

Alro and Skovsmose's ideas show how understanding and issues of social justice are connected in important ways. The idea that there are ways in which technical elements of understanding in teaching and learning situations are connected to issues of equity and social justice is a central concern of the analysis in this study. The theoretical foundations of complex instruction have a strong focus on equity and teaching all students to high standards, but they are primarily concerned with teaching using groupwork with heterogeneous classes (Cohen 1994). What is the technical connection between ideas like mixed ability setting (or heterogeneous classrooms) and effective teaching and learning from a point of view of communicative action? Further, the research on the effectiveness of these approaches in the mathematics classroom suggests that equitable teaching techniques are responsible for greater participation and attainment compared to traditional approaches (Boaler 2006; Boaler & Staples 2008; Boaler & Staples 2008).

Kilpatrick, Biehler, Skovsmose and Hoyles discuss issues of meaning, collective meaning, the reconstruction of meaning as a didactical task, the role of common sense in collective meaning, and the communicative construction of meaning in the book *Meaning in Mathematics Education* (Kilpatrick et al. 2005). An interesting concept that may have connections to Habermas' Theory of Communicative Action is the notion of common sense. In the aforementioned book, Kilpatrick et al. address the issue of common sense in "contrast and interplay with science":

> Common sense suggests that to know means to justify conclusions that are already formed, whereas in science, to explain means to establish a synthetic relation between premise and conclusion. (ibid. p4)

They go on to suggest that science, though contrasted against common sense, also depends on it in a dialectical fashion and that common sense is part of the problem of collective sense-making. This is a very interesting idea that has resonance in Habermasian concepts of intersubjectivity. In particular the following claims can be supported by Habermas' arguments in the TCA: 1.) common sense can be conceptualised as a way in which intersubjectivity presents itself; and 2.) the theory of communicative action can be conceptualised as the describable rationality of communication that presents itself as intersubjectivity and common sense.

These claims, based in the TCA, have clear connections to the work done by Kilpatrick et al., and suggest that there may be potential complementarity between the arguments made by those authors and arguments based in the TCA. As always with the consideration of the relation of theoretical works not based explicitly in the same literature, there is a need to be also aware of theoretical differences and conflicts as well as similarities so as to give each its due and not reduce one to the other in an unwarranted fashion.

Another interesting line of research and theory development in the effort to understand interactions and intersubjectivities in mathematics education is the work of the self-described interactionists. As has been touched upon above, researchers such as Cobb, Bauersfeld, Steffe, Voigt and others developed a theoretical framework that has sought to move beyond radical constructivism, to a position that might more appropriately be called a social constructivist position, in that it seeks to maintain the use of constructivist ideas about mathematics education, while incorporating theories of the social that are not merely founded in radical constructivism. Bauersfeld, (1994a), outlines the core convictions of the interactionists as follows:

> 1. *Learning* describes a process of personal life formation, a process of an interactive adaptation to a culture through active participation (which, in parallel, reversely constitutes the culture itself) rather than a transmission of norms, knowledge, and objectified items.
>
> 2. *Meaning* is with the use of words, sentences, or signs and symbols rather than in the related sounds, signs, or representations.
>
> 3. *Languaging* describes a social practice (the French *parole*), serving in communication by pointing at shared experiences and for orientation in the same culture, rather than an instrument for the direct transportation of sense or as a carrier of attached meanings.

> 4. *Knowing or remembering something* denotes the momentary activation of options from experienced actions (in their totality) rather than a storable, deliberately treatable, and retrievable object-like item, called knowledge, from a loft, called memory.
>
> 5. *Mathematizing* describes a practice based on social conventions rather than the applying of a universally applicable set of eternal truths; according to Davis and Hersh (1980), this holds for mathematics itself.
>
> 6. (Internal) *representations* are taken as individual constructs, emerging through social interaction as a viable balance between the person's actual interests and realised constraints, rather than an internal one-to-one mapping of pre-given realities or a fitting reconstruction of "the" world.
>
> 7. Using *visualizations and embodiments* with the related intention of using them as didactical means depends on taken-as-shared social conventions rather than on a plain reading or the discovering of inherent or inbuilt mathematical structures and meanings.
>
> 8. *Teaching* describes the attempt to organise an interactive and reflexive process, with the teacher engaging in a constantly continuing and mutually differentiating and actualizing of activities with the students, and thus the establishing and maintaining of a classroom culture, rather than the transmission, introduction, or even rediscovery of pre-given and objectively codified knowledge. (ibid. p138)

Again there are clear potential connections to a theoretical framework based in Habermas' Theory of Communicative Action. The question then might be whether the theory of communicative action has anything to offer in addition to the already established theories within mathematics education. This is a legitimate question, and a large part of this thesis will be spent in making the argument that they do. In particular one can see emerging a potential lack of specificity in the consideration of the form of the processes of intersubjectivity, and it is precisely these issues that Habermas' theories can potentially address with their conceptualization of the preconditions for intersubjectivity and communication. A theoretically coherent analysis of evidence of intersubjectivity from a Habermasian standpoint might be a foundation upon which the critical potential of Habermas' project could be brought to bear on issues directly concerning mathematics education.

In formal educational situations learning can be seen as a social activity which takes place in the context of a community of learners and practitioners and may be construed as the process of becoming a member of a community of practice (Lave & Wenger 1991). Interactionist, situated, and socio-cultural perspectives of mathematics education provide a useful framework for the analysis of the complex dynamics of classroom practice. These ideas seek to go beyond the limitations of psychological interpretations of learning, teaching, and understanding as an experience of the individual by considering the linguistic and social contexts in which meaning is constituted. This move reflects a recognition that learning is social, which is an essential counterpoint to radical constructivist epistemologies of mathematics as it emphasises the vital role that discourse and context play in the development of conceptual understanding. This intellectual movement within mathematics education also takes into consideration the nature of social phenomena from a more than psychological perspective of schema building. The interactionist position is an attempt to take these epistemological issues (that of the nature of scientific knowledge and the nature of social phenomena as the object of social science), and integrate them into the research from a number of different perspectives. These varied perspectives include that of sociolinguistics in the UK, of Vygotskian socio-cultural theory, and of theories of cognitive development based on Piaget's ideas (Cobb & Bauersfeld 1995).

This social turn is complemented by semiotic analysis of student discourse in mathematics classrooms and consideration of implications for pedagogy and curriculum (Cobb et al. 2000). These authors bring up and attempt to resolve issues arising from the dynamic nature of making meaning through discourse. There are many different perspectives about the relationship between action, experience and meaning put forth in *Symbolizing and Communicating in Mathematics Classrooms* (ibid.). One of the interesting notions that seems to grasp the connection between the multiple theoretical views and their practical implications is in the chapter by Nemirovsky and Monk where they suggest that a central issue for design is the creation of environments that are conducive to mathematical meaning making without being deterministic of exactly how the meaning will emerge and what form it will take. In the later half of the book, the focus changes from more theoretical analyses to implications for design. One of the interesting ideas discussed is that of Distributed Support for Knowing.

This idea suggests that the expertise in the sense of being able to perform a task is often distributed over a number of different people and artefacts and that this can be taken into explicit consideration when designing learning environments by having students work collaboratively to invent artefacts to help them achieve tasks more efficiently. In this chapter there is also an interesting discussion about the types of norms that might be conducive to the collaborative and deeply mathematical activity that was envisaged by the authors. Norms like expecting students to take risks and generate new ideas were seen as important but insufficient with regard to content specific objectives. The need for specific connection of normative behaviour to the deep mathematical concepts and practices was seen as essential. Thus the norm of generating new ideas might be combined with discussions on the importance of developing useful mathematical artefacts so that the kinds of ideas that are generated are focused on underlying conceptual features of the discipline rather than on surface features. Other normative aspects covered in this section of the book include ways to build interdependence into the learning environment either through normative means (such as group and individual accountability structures) or through task design features such as 'jig-saw' style lessons where students first work to develop artefacts collaboratively and then break up and teach other groups about their artefacts (Cobb et al. 2000). In many ways this work and the work that has been based upon it in more recent years provides the theoretical foundation for research and analysis of discourse, communication and collaborative work in mathematics education.

In addition to these perspectives there is a whole literature of discourse in mathematics education that has been developed over the last 10 to 20 years. A recent article in The Journal for Research in Mathematics Education (JRME) by Ryve (2011) undertakes a fairly comprehensive analysis of a large part of the articles that make up this emerging research trend. Broadly defining the issue of research into discourse as research about communication, Ryve addresses a number of questions in relation to the articles that he undertakes to analyze including how and to what extent the articles are theoretically conceptualised, what data are used and how are the data analyzed, and to what extent do the articles relate to or build upon one another? In his analysis Ryve notes:

> This study shows that a wide variety of theories and approaches are imported from traditions outside mathematics education to examine aspects from all three topic areas: Social Interaction; Mind, Selves, and Sense-making; and Cultural and Social Relations. On the other hand, the study also indicates that the conceptual clarity of many articles is weak and that cumulative work of developing theoretical approaches for conceptualizing and analyzing discourses is rare. One may argue

> that conceptual clarity is a prerequisite for cumulative development because it is very difficult to build sensibly on other articles if keywords and/or epistemological principles are not explicitly discussed. (Ryve 2011)

This analysis seems to reinforce my own thoughts about how to integrate the use of multiple theories in a theoretically coherent manner. It also relates to the issues that are raised earlier in this chapter by Lerman with regard to the theoretical coherence in the treatment of intersubjectivity. Bussi (1994) addresses this issue in her notion of the necessity of 'complementarity' as a feature of the use of theories and the need to identify conceptual tools to handle the tension between and integration of use of multiple theoretical perspectives as outlined in her chapter 'Theoretical and Empirical Approaches to Classroom Interaction' in *The Didactics of Mathematics as a Scientific Discipline* (Biehler 1994a). I will return to this idea in the final section of this chapter and suggest a methodological approach that is based in Habermasian concepts of the development of theory in the social sciences. Ryve goes on to make two further claims in his results that are pertinent to this research; they concern the implications of the diversity of theories found in addressing issues of communication in the literature in mathematics education. First is his assertion that the priority should be on developing the sophistication of the theoretical perspectives that have already been developed rather than introducing further approaches from other fields.

> These results suggest that it is more important for future studies in mathematics education to engage in theoretically sophisticated development of already-introduced theoretical approaches than to import new approaches from other fields. (Ryve 2011)

This raises for me the question of whether I am engaged in one or the other of these approaches. Is the use of Habermas' Theory of Communicative Action the introduction of a new theoretical approach, or is it rather the use of theory which is conceptually related to multiple theoretical approaches already in use in mathematics education in an attempt to develop the sophistication of approaches to understanding communication in mathematics education? I make the case that to a large degree it is the latter, and that, through the use of ideas like Bussi's concept of complementarity, ideas and analysis based in the TCA can be used productively to illuminate aspects of practice and learning that are already conceptualised from theoretical perspectives that do not explicitly make use of the TCA. Finally, Ryve address the detailed aspects of theoretical sophistication that need to be developed in research on discourse:

> The results indicate that general features of theoretical development such as defining keywords, building on the work of others, and clearly positioning the article in epistemological perspectives are of great importance for future studies in mathematics education. (ibid.)

One of the theories to which I relate the use of the TCA is that of Situated Theory. Lave and Wenger developed the ideas of Situated Learning in the early nineties and it has gained widespread acceptance and use as a productive theoretical perspective for the analysis of learning and teaching in mathematics education. Some of the interesting ideas that Lave and Wenger include are the concept of communities of practice as the location of identity formation and learning, the concept 'legitimate peripheral participation' as a process of coming to belong to such communities, and learning as the process of forming an identity in the context of a community of practice (Lave & Wenger 1991). They address discourse in this work by suggesting that in the context of apprenticeship in a community of practice the notion of legitimate participation is key to understanding the role of language in learning and forming an identity in the context of a particular community of practice. These ideas taken alongside some of the issues raised in Wenger's doctoral thesis, particularly the role of institutions and their parasitic relationship with the meaningful activity of communities of practice, the interactional basis for the development of meaning in communities of practice, and the role that communities of practice play in the production and reproduction of the social, seem to indicate the potential for theoretical complementarity with ideas that form the basis of Habermas' TCA (Wenger 1990).

Greeno and *The Middle School Mathematics Through Applications Project Group* (MMAP) (Greeno et al. 1997) research use situated theory to re-conceptualise the way in which learning mathematics takes place and bring to the fore the issue of participation, practices, the concept of an ecology of learning, and the new ways in which agency and identity can be thought of in the context of mathematics education. Boaler has also been in the forefront of using situated theory to analyze mathematics learning and teaching, beginning with her work in England and following on in her work in the US including some collaborative pieces with Greeno (Boaler 1999; Boaler 2000a; Boaler & Greeno 2000). This literature can serve as an important reference for the use of situated theory in the analysis of evidence of intersubjectivity and may complement it by examining in detail how the ideas of situated theory relate to the technical expression of the preconditions for communication and intersubjectivity in complex instruction style teaching and learning.

Further, Cobb also raises the importance of situated theory to the ideas of the interactionist school in his article in *Multiple Perspectives on Mathematics Teaching and Learning*, arguing for a pragmatic use of situated theory that rejects a purely psychological point of view in the attempt to formulate theory that can have practical application in the improvement of mathematics education practices by taking into account theories that address the social aspects of learning (Cobb 2000). Taking these points together it seems clear that there may be a way to use Habermas' TCA in a complementary and coherent manner in relation to at least Situated Theories and possibly other theories as well. However, it is also clear that this is far from a non-problematic proposal and that care must be taken to position the theories in relation to each other and be clear about the limits of these attempts at complementary use of theory in analysis.

2.4 Groupwork, complex instruction, and reform oriented mathematics education

In the teaching of mathematics there is a generally accepted spectrum of pedagogical approaches. This spectrum ranges from traditional rote learning with little or no emphasis on conceptual understanding or complex problem-solving skills to reform oriented learning that aims to develop understanding by incorporating meaning making and investigation in mathematics teaching and learning. Traditional approaches to teaching mathematics concentrate primarily on rote learning. This pedagogical strategy is characterised by a focus on instrumental understanding with little or no attention given to the development of relational understanding (Skemp 1971; Boaler 2002). It is also often characterised by a focus on producing the one right answer to a problem and the role of the teacher as the primary legitimating authority.

In contrast to traditional methods, reform approaches are characterised by a focus on the socially constructed nature of mathematical knowledge and can be explicit about the part played in this process by both teachers and students. With this focus comes an emphasis on mathematical discourse, complex problem-solving, and mathematics in context. The US-based Curriculum and Education Standards for School Mathematics (NCTM 2000) is widely regarded as a description of agenda of reform mathematics with its focus on communication, problem-solving, and conceptual understanding. Complex Instruction is a pedagogical approach that combines a curriculum focused around open-ended mathematical tasks central to the discipline of mathematics, instructional strategies that foster collaborative groupwork and problem-solving, and a re-conceptualization of competency so that all students have a chance to contribute meaningfully and thus gain the skills, knowledge and confidence to explore the rich domain that is mathematics (Boaler 2006; Boaler 2008). This approach is firmly in the realm of reform-oriented practices and is the one which practitioners in this research attempted to adopt, thus forming the context for the research in this thesis. In the same vein, related research and issues regarding groupwork and teacher learning and practice form an essential contextual element for this research. Beginning with the US-based NCTM standards document in 1989, and elaborated in the revised Principles and Standards document in 2000 (NCTM 2000), the notion of Reform oriented mathematics practices gained ground.

2.4.1 Groupwork

'Massively Collaborative Mathematics' is the title of Jordan Ellenberg's contribution to 'The Year in Ideas' in the *New York Times Magazine* of Thursday 10th December 2009. This article reflects on the successful collaborative attack on a stubborn mathematics problem orchestrated by Timothy Gowers, a Cambridge Mathematics Professor and holder of the prestigious Fields Medal, in the comment thread of his blog. It is in the context of profound new developments in mathematics collaboration and communication at the highest levels such as these that one may consider the need for mathematics education to continue to grapple with the inertia and intransigence of traditional mathematics pedagogies. Significantly, one of the decisions made by Gowers was to establish a set of norms for the collaboration from the outset - a significant connection to this research especially given the startling success of Gowers' project. It seems clear that mathematics education must be concerned with providing students with the kinds of experiences which will not only give them an equitable and effective mathematics education but which will also provide them with the knowledge and practices necessary to contribute in new collaborative spaces. This review seeks to illustrate some key aspects of the thinking and research that has been conducted in the mathematics education research community around understanding not only the theoretical aspects of discourse, collaboration and groupwork in mathematics classrooms but also the practical aspects of fostering learning environments and experiences conducive to productive discourse and collaboration.

There is rapid growth in awareness and application of ideas in educational technology literature, with the notable example of Stahl's work on CSCL. Gary Stahl has developed theories of group cognition in the context of computer supported collaborative learning (CSCL) that bear closer examination. Not only does Stahl have extensive background in the philosophy of language, but he integrates this into his analysis of the many research projects in CSCL that he has conducted over the course of his career. Among the issues that Stahl covers in his theory of group cognition are: Building Collaborative Knowing; Group Meaning versus Individual Interpretation; Shared Meaning, Common Ground, Group Cognition; Making Group Cognition Visible; and Thinking at the Small-Group Unit of Analysis (Stahl 2006).

Another interesting line of work is the development of an analytical framework involving 'focal and preoccupational analysis' to analyze mathematical discourse in small groups (Ryve 2004). Ryve investigated whether groups of engineering students communicated effectively and analyzed conversations to find characteristics of mathematically productive discourse. Ryve's work is explicitly founded on the theories of discursive psychology that are developed in the work done by Sfard (2001), specifically the idea of communication as cognition. Ryve's paper is useful as an example of how to apply the semiotic theories, which have been developed to a study of learning in action. Educationally productive discourse is characterised as interactions that have an impact on students' participation in future related mathematical problem-solving activity (Kieran 2001). Focal analysis contributes to the analytical framework by addressing the different uses and meanings that students may attribute to the same symbols or utterances, while preoccupational analysis contributes by addressing the interactions and communication between participants in discourse (Sfard & Kieran 2001). An interesting technique used in this study was the use of an interactive flowchart to facilitate the preoccupational analysis of discursive moves between the participants.

More recent developments in semiotics include Morgan's work on Social Semiotics, which offers a methodological development to the semiotic work done in the field in the form of tools for the analysis of texts in mathematics classrooms. The major contribution to the theoretical domain is the inclusion of "…Halliday's theory of language as social semiotic

(Halliday, 1978; Halliday & Hasan, 1989) and the associated tools of systemic functional linguistics (Halliday, 1985)…" (Sfard & Kieran 2001). Brodie has also contributed to the area of study with regard to small-group work in the field of mathematics education with her two articles in *For the Learning of Mathematics* examining teacher intervention (Brodie 2000) and more recently, her consideration of the ways in which teachers structure conversations in mathematics classrooms (Brodie 2007). Dekker and Elshout-Mohr also examined teacher intervention in the context of collaborative mathematics learning and found that interventions focused on supporting the interactions of the students were more beneficial than interventions focused solely on the mathematical content (Dekker & Elshout-Mohr 2004). Another study by Pijls, Dekker, & Hout-Wolters, found that students who engage in explanation and critical reflection during collaborative mathematics learning make greater gains than participants that do not engage in these activities (Pijls et al. 2007).

A recent review of research on teacher practices in the implementation of pedagogy focused on mathematical communication. An interesting aspect of this review is the use of Activity Theory to analyze and understand the evidence in the research literature. This approach could mesh well with the use of Habermas' theory of communicative action, but would again need a brief positioning of the theoretical traditions to one another. As part of their findings the authors make two pertinent arguments that relate to this research. First, addressing the need to examine the evidence of practice in order to understand how communicative contexts can be created and supported in mathematics classrooms:

> Finding out what kinds of contexts and communities support mathematics discourse for outcomes-focused pedagogy is crucial for education. Teachers who implement pedagogical reform, in relation to classroom discourse, must inevitably focus on developing community, ensuring that those within the community are given opportunities to talk about, support, and nurture each other's learning. The review we have provided here represents systematic and credible evidence. (Walshaw & Anthony 2008)

The second main set of findings is pertinent to this research in that the issues of respect and inclusion and the relationships of these to multiple aspects of mathematics learning find resonance in concepts of the preconditions for communication and intersubjectivity contained in the TCA:

> We found that inclusive classroom partnerships are fundamental to effective teaching. Facilitating respectful and patterned interactions in the classroom contributes to the enhancement of students' aspirations, attitudes, and achievements. Teachers who set up conditions that are conducive to classroom discussion come to understand their students better. Students benefit, too, and the ideas put forward in the classroom become rich resources for knowledge. Through students' purposeful involvement in discourse, through listening respectfully to other students' ideas, through arguing and defending their own positions, and through receiving and providing a critique of ideas, students enhance their own knowledge and develop their mathematical identities. (ibid.)

There is much research on groupwork not specifically focused on mathematics education that might also be useful, as can be seen in one particular special issue of the Cambridge Journal of Education on group work that was published in 2009. It included topics such as the connection between groupwork and communication, enhancing problem-solving and argumentation, and empirically based understandings of collaborative reasoning (Webb et al. 2009; Reznitskaya et al. 2009; Kutnick & Berdondini 2009; Gillies & Khan 2009; Galton et al. 2009; Galton & Hargreaves 2009; Christie et al. 2009; Baines et al. 2009).

2.4.2 Complex instruction

The importance of Complex Instruction is that it explicitly deals with some of the issues raised by the social and linguistic turns in mathematics education research and also the fact that some interesting results in mathematics education have been made in the context of classrooms using complex instruction in mathematics. Among these results are Boaler's studies of Railside School in California (Boaler 2003; Boaler 2008; Boaler 2006; Boaler & Staples 2008). Complex Instruction is a pedagogical approach that combines a curriculum focused around open-ended mathematical tasks central to the discipline of mathematics, instructional strategies that foster collaborative groupwork and problem-solving, and a re-conceptualization of competency so that all students have a chance to contribute meaningfully and thus potentially develop more positive identities as learners of mathematics and learn more (Boaler 2006; Boaler 2008). There are four main elements of the approach: 1) the use of 'skill-builder' activities that focus on developing students collaborative practices; 2) the development and use of 'group-worthy tasks' (Lotan 2003) that are designed to promote interdependence and provide multiple opportunities for developing understanding; 3) Classroom 'norms' that promote collaboration (like 'nobody's done until everybody's done' and 'Everyone Helps') and 'Roles' that seek to delegate authority for learning to the students by assigning specific responsibilities to different students[12]; and 4) teacher interventions to promote equitable and high rates of participation. Initial research by the founders of the approach suggested that there was a balance between structure and autonomy that needed to be navigated in order for students to benefit from interacting academically with their peers (Cohen 1994; Cohen & Lotan 1997; Cohen et al. 1994). This analysis is described well by Staples who put forward complexity theory as a conceptual framework to deal with the open questions of how to handle the seemingly contradictory demands of structuring groupwork to promote productive collaboration (Staples 2008).

2.5 Teacher learning and practice

The importance of issues of teacher learning and practice in the context of this research has to do with the fact that the sites chosen where adopting new practices, specifically mixed ability teaching using complex instruction approaches. Knowledge of the nature and the challenges of teacher learning in the context of adopting alternative approaches to teaching mathematics are addressed in Deborah Loewenberg Ball's 1994 report on teacher learning. One of the fundamental issues with regard to supporting teachers' adoption of alternative practices is that the nature of the new reforms in mathematics education is based on the ideas of fallibilism and constructivism. These ideas about mathematics education challenge received images of teaching and of the nature of mathematics knowledge (Ball 1994).

Ball illustrates the difficulties that elementary school teachers face when confronted with alternative approaches to mathematics teaching, and notes the prior experiences that elementary school teachers bring which shape their learning. She elaborates on the idea that shortcomings in teachers' own maths knowledge are not to be blamed on faulty school mathematics teaching that they themselves have experienced but often on personal inadequacies. This is symptomatic of one of the major challenges facing the wider adoption of alternative approaches to mathematics education. Teachers are the main agents for this type of educational change as it is their classroom practice which must shift. As products of the system in which they become the main agents of change they face the particular

[12] e.g. http://nrich.maths.org/content/id/6971/RolesKS1.doc

challenge of overcoming ideas about teachers' roles, who can learn mathematics, and what it takes to learn mathematics.

While Ball details the challenges for elementary school teachers who are not confident about their knowledge of mathematics, similar challenges face teachers at all levels who are more confident about their knowledge (Ball 1990). These teachers have formed identities within a system that has certain values about who can and who cannot do maths, what mathematics is and how it should be taught and learnt. Teachers who were successful students are likely to be influenced by these experiences and hold beliefs and practices that are in line with these traditions. The challenge presented by this is that it is precisely these traditional ideas about what mathematics is and how it ought to be learnt that need to change in order for alternative practices to be adopted successfully.

One of the insights from the US experience of adopting alternative ways of teaching mathematics is that it entails a shift in direction from a naive confidence in the unproblematic nature of mathematics teaching and learning towards a more honest confrontation of the uncertainties with regard to the nature of learning and understanding the subject. Some significant things known about teacher learning include the idea that teaching alternative approaches to mathematics is hard to learn how to do and takes time. It seems reasonable to assume that this has to do with paradigmatic shifts required of teachers of mathematics and the fact that alternative approaches to mathematics teaching put new demands on teachers with regard to the interplay between pedagogical content knowledge and subject knowledge.

There is a short list of practices that have been identified as useful in the context of adopting new teaching practices. These include staff development where there is follow-up with others involved in the same kind of learning/change. Modelling practice in professional development (as in IMP[13] training sessions, etcetera) is a useful strategy. A focus on subject matter knowledge is important in teaching for understanding. Managing classroom discourse around complex concepts can be difficult if the teacher is not confident about the mathematics. Subject matter knowledge can help to interpret alternative solutions and propositions put forth by students. Finally, while it has been shown that reflection is important in supporting the growth of practitioners, this idea needs more focus if it is to be helpful with regard to mathematics education (Ball 1994). These

[13] 'Interactive Mathematics Program'

insights into the nature of teacher learning with regard to the adoption of alternative approaches to the teaching of mathematics provide useful elements of a framework for considering the challenges and the experiences of teachers in the adoption of complex instruction pedagogies in mathematics classrooms.

2.5.1 Beliefs and practice in mathematics education

The study of the impact of adopting non-traditional mathematics pedagogy with regard to teacher beliefs and practices must take into consideration the research that has already taken place concerning similar situations. Some of the most interesting and extensive research into these phenomena comes from the beginning of the mathematics reform movement in the United States also referred to as the Standards Based Reform Movement (Gardner et al. 1983). The values of this movement were widely publicised and began to gain significant momentum in mathematics education with the release of the 1989 Curriculum and Evaluation Standards for School Mathematics from the NCTM. However, these standards were not completely unexpected but rather reflected the current state of practice in a significant part of the mathematics education community.

Beginning in this early context, a series of studies and research papers were produced by several mathematics education researchers to examine the relationship between teachers' beliefs and their classroom practices (Ball 1988; Ball 1994; Boaler 1998; Boaler 1999; Sztajn 2003; Remillard & Bryans 2004). This ongoing course of research occurred in part because it was recognised early on that part of the challenge of trying to enact these kinds of reform had to do with the different demands put on teachers. The predominant recurring theme in the research literature suggests that it is an ongoing problem as the mathematics education community continues to integrate alternative approaches to teaching into classroom practice. Teachers were being asked to think and act in the mathematics classroom in ways which were unfamiliar to them and that did not form the basis of their experience as learners of mathematics (Ball 1988).

Part of the challenge of adopting new practices in mathematics education has to do with the personal educational experiences that teachers bring with them into the profession. This conceptualization of the challenge is based in constructivist theories of education that see the understanding teachers have about how to teach and how students learn mathematics as shaped in large part by what they bring to their teacher training and their practice from their prior experiences. In this regard the research is clear, that teachers' beliefs and experiences play a large role in their own learning (Ball 1994).

Teachers teach the way they have been taught (Lortie, 'the apprenticeship of observation') because the preponderance of their own experience outweighs the examples of alternative practice and theory to which they have access in their teacher education courses (Lortie 1975). Ball explored this issue of breaking the conservative cycle in the context of a teacher education course which she implemented. The 'Exploring Teaching' class was focused on raising questions about teaching, and examining commitment to teach, as well as preconceived ideas about teaching and learning and learning to teach. Feelings about oneself in relation to mathematics affect teacher attitudes towards mathematics. Many elementary teachers feel insecure in their knowledge of mathematics and studies suggest that much of what they know about mathematics is procedural (Ball 1994; Ball 1988). The case may be radically different with regard to secondary teachers who have often achieved a modicum of success in learning mathematics in traditional settings.

Research suggests that teachers' beliefs about who can learn affect how they teach and how they perceive students' performance (Ball 1988; Boaler 2002; Sztajn 2003). For instance, when teachers rationalise the failure of their students it tends towards a self fulfilling prophecy; alternatively, when teachers defy these rationalizations and teach students with the belief that they can learn if given the opportunity, then there are more positive results. This is described as a 'pedagogy of poverty' (Haberman 1991).

Ball outlines a series of ideas that form key elements of at least part of a conceptual framework in the area of beliefs and practices in the adoption of nonstandard teaching practices. One of the first ideas that, for me, leapt from the page in Ball's argument is the descriptive claim that 'there is an image of mathematics as passing on knowledge, which teachers need to learn to move beyond'. Despite the fact that much of the content of mathematics education courses is about how to teach from a more or less constructivist perspective, it is often the case that teachers end up teaching as they were taught due to a number of political social and professional pressures (Ball 1988). Ball ascribes the idea that "learning to teach requires overcoming the limits of their firsthand experience" to Buchmann and Schwille (Buchmann & Schwille 1983). Ball argues that much of mathematics teaching is pedagogically naive in that many mathematicians have taught as though clearly stating the necessary propositions and then articulating the form of the mathematical argument with attention to precision is sufficient for it to be understood by their students. This makes the mistake of conflating what is useful for the understanding of one who has mastered the material and what is useful to those who are still in the process of learning that material. This idea has resonance with ideas from Vygotsky about the zone of proximal development (Wertsch 1986).

2.5.2 Pedagogical content knowledge

Given the explicit focus on pedagogical factors encountered by teachers adopting complex instruction approaches in mathematics teaching, it is vital to consider the idea of pedagogical content knowledge as put forth by Shulman, and how it relates to the adoption of complex instruction pedagogies in mathematics teaching (Shulman 1986). Many of the new opportunities and demands associated with the adoption of complex instruction in mathematics education are closely related to pedagogical content knowledge. The distribution of authority and the focus on students engaging in collaborative problem-solving and conceptual development emphasises the teacher's role as a strategic coach of individuals and small groups and as a facilitator of conceptual whole class discourse. In order to promote appropriate conceptual development the teacher must be able to observe and assess the students' thinking and respond to it by engaging them in developmentally appropriate discourse focused on, the conceptual content of their work, the tasks at hand, and the way in which these relate to the wider domain of the discipline.

Such pedagogical content knowledge is not independent of disciplinary content knowledge, pedagogical knowledge, or curriculum knowledge. Rather it is an essential category of teacher skill and action in which the teacher mediates the interaction between the content as represented by the curriculum, the teachers' understanding of the curriculum and the understanding of the student as expressed in their written work and discourse. However, the changing practice of teachers in the adoption of complex instruction in mathematics will find much of its expression in the domain of pedagogical content knowledge and it is here that many of the challenges and opportunities of adopting such practices may be found.

2.6 Sociological Theories addressing the relationship between macro-level analysis and micro-level analysis

While Habermas deals with education in the second volume of the Theory of Communicative Action (Habermas 1985b), there are other sociologists of education who also deal with the macro micro level dialectic. Could the research and findings in this thesis, which are primarily focused on microanalysis of small group interactions (based in Habermas' theory of communicative action), inform a wider sociological analysis of the relationships between micro features of classroom practice and macro features of institutions and culture? I will briefly address the approaches and potential of several other theorists, before making the argument that it is appropriate in this thesis to focus primarily on concepts and theories based in Habermasian critical theory. While drawing on these multiple theoretical approaches simultaneously may be possible, it is beyond the scope of this thesis, particularly due to the extensive theoretical networking efforts that might be required (Radford 2008) and also due to the explicit focus in this work on the microanalysis of small group interactions using theory and models based in Habermas' theories. Future work may benefit from an approach which is informed by critically drawing on a wider range of sociological perspectives and bringing them into relation with one another in order to more fully explore the implications of findings of this thesis, which are based primarily in Habermas' ideas (which are already based in a critical reconstruction of sociological perspectives including Weber, Mead, Durkheim, Parsons and Marx).

The educational sociology of Bernstein, Bourdieu and Foucault, along with Habermas, see education as playing a central role in the reproduction of society and thus notably the inequitable relations within society. Bernstein's theories of codes and class, pedagogic control and symbolic power conceive of the discourse happening in the classroom as reproducing the class structure through the exercise of power (Bernstein 2000). One relevant issue here is how power at the micro level influences and/or relates to the cultural reproduction-production at the macro level.

> In other words, the notion of pedagogic practice which I shall be using will regard pedagogic practice as a fundamental social context through which cultural reproduction-production takes place. (Bernstein 2000, p3)

In retrospect, these theoretical resources could complement further elaboration of the kinds of systematic distortions of communication identified in the analysis in this thesis. This could allow for a better understanding of the relationship between features of society at both a cultural and a systematic level and the conflict and use of power in micro-analytic settings of the classroom.

Foucault has a slightly different take on the role of education, seeing it from his perspective of historical analysis as a central example of the use of power in modernity to enforce social control through observation. Foucault addresses the concept of examination in the context of the developments of education in modernity:

> The examination combines the techniques of an observing hierarchy and those of a normalizing judgment. It is a normalizing gaze, a surveillance that makes it possible to qualify, to classify and to punish. It establishes over individuals a visibility through which one differentiates them and judges them…The superimposition of the power relations and knowledge assumes in the examination all its visible brilliance. (Foucault 1995, pp184-185)

This concept of how practices at the cultural and institutional level play out on the local level to create and reinforce the conditions of modernity mark a powerful analysis of the ways in which the social reality of modernity is constituted and could play an important role in future analysis of the complex ways in which power shapes the development of knowledge and practices in the classroom. In particular in Foucault's analysis the school becomes a central institution in the exercise of power on the constitution of society:

> Similarly, the school became a sort of apparatus of uninterrupted examination that duplicated along its entire length the operation of teaching. It became less and less a question of jousts in which pupils pitched their forces against one another and increasingly a perpetual comparison of each and all that made it possible both to measure and to judge. (Foucault 1995, p186)

Foucault's theories and analysis offer an important critique of modernity and its institutions, highlighting its historical contingency and the temporal processes which form and give rise to the features of modernity which might otherwise have been taken for granted. Further, it may provide insights into the way participants are constituted in relation to power, as Foucault indicates this historical analysis is characterized by "the 'formalization' of the individual within power relations." (ibid., p190) The extensive nature of Foucault's work in regards to the ways in which discourses within modernity can be analysed to develop insights into just such power relations, means that it could be of use in examining the analytical issues that arise in this thesis.

However, it was not initially clear how these ideas could be used in the development of micro-analytic interpretations of interactions such as those which are the primary focus of this work. While the analysis in this thesis came to be focused on the use of Habermas' theories, further analysis and future work based on this research may benefit from consideration of Foucault's work in the development of analytic themes which might be broadly considered to address the macro/micro divide in sociology. The use of Foucault's (2002) methods of analysis is not a trivial task, yet it could be productive to consider in future work which seeks to draw on a wider range of sociological theory in building on the findings from the research and findings in this thesis.

Bourdieu's sociology also features a theoretical treatment of how inequality is reproduced in society (Swartz 1997; Bourdieu & Wacquant 1992). In particular his idea of symbolic violence and the associated idea that all symbolic systems are forms of power might contribute to analysis of how some of the issues of power that arise in the microanalysis of interactions in this thesis relate to wider issues of the social reproduction of inequality, particularly in the context of mathematics education. It is also interesting to note that Bourdieu's theory explicitly attempts to challenge the idea of a macro/micro divide in social theory with his inter-related concepts of field and habitus (ibid).

Habermas, however, addresses the macro micro divide in a very particular manner by reconstructing a theory of the lifeworld based in a re-conceptualisation of intersubjectivity and the nature of communicative action in the reproduction of the lifeworld. He then critically engages with Parsons' system theory to show how the macro systems of society depend on the communicative action and intersubjectivity of lifeworldy practices. These issues address the reproduction of society at various levels:

> Under the functional aspect of mutual understanding, communicative action serves to transmit and renew cultural knowledge; under the aspect of coordinating action, it serves social integration and the establishment of solidarity; finally, under the

> aspect of socialization, communicative action serves the formation of personal identities. The symbolic structures of the lifeworld are reproduced by way of the continuation of valid knowledge, stabilisation of group solidarity, and socialisation of responsible actors….Corresponding to these processes of cultural reproduction, social integration, and socialisation are the structural components of the lifeworld: culture, society and person. (Habermas 1985b, pp137-138)

The above quote needs to be understood as dealing with primarily the reproduction of the symbolic structures of the lifeworld. Habermas also deals with the material reproduction of the conditions for the lifeworld, which he see as the result of goal directed action of 'sociated individuals'. The system is seen as fundamentally distinct as it based primarily in strategic and instrumental action as opposed to communicative action. Thus, politics, government and the economy coordinate action not through the medium of mutual understanding, but rather through the dual media of power and money (though it should be noted that Habermas (1985a, 1985b) also argues that strategic action has a derivative relationship to communicative action).

Habermas' further develops a theoretical argument that suggests that the instrumental and strategic reason enshrined in institutional systems can encroach on and distort the communicative action in the lifeworld (the functioning of the micro structures) and thereby threaten the integrity and vitality of the 'systems' of society (the functioning of the macro structures). This is addressed elsewhere in this thesis (Chapter 2 Section 2.1.5), but is noteworthy in this section as it indicates that there is potential to pursue sociological questions based in this research at more macro levels using Habermas' critical theory.

One aspect of the rationale for drawing primarily on Habermasian theories to support the analysis in this thesis is to avoid, at least in the first instance, the pitfalls of using multiple theoretical perspectives in an uncritical manner. While each of the authors mentioned in this section deal with macro/micro issues, they all deal with these in ways which are sometimes slightly and sometimes significantly different. Consider Swartz's interpretation of Bourdieu's position on the nature of rationality in modernity in relation to Habermas and Foucault:

> One can understand Bourdieu's reasoning here as a positioning strategy that gives him leverage against Foucault and deconstructionists like Derrida at one extreme

> and Habermas at the other. Vis-à-vis the post-structuralists, Bourdieu affirms the method and norms of the Enlightenment tradition of rationality. He sees this tradition as offering a form of knowledge that is self-referential and capable of some degree of self-transcendence…. Vis-à-vis Habermas, however, Bourdieu associates the claim for a transcendental reason with an interested position within the intellectual field. But it is also a type of interest that Bourdieu wants to institutionalize and develop. (Swartz 1997, pp252-253)

This thesis began with the microanalysis of small group interactions and developed, through a theory seeking (or explanatory) case study, a set of codes and models to understand small group interactions and the critical potential therein, from a perspective firmly based in Habermas' theory of communicative action. Therefore it is only reasonable, consistent, and analytically coherent to explore macro/micro links from the same theoretical perspective, at least to the extent that they are in the current work. In conclusion, while the theory and interpretations in this thesis were developed with the intent that they might serve as a basis for exploration of wider features of the social reality of mathematics teaching and learning, that work, and associated potential engagement with a wider range of sociological thought, which might require networking of Habermasian theoretical perspectives with other (potentially complementary) theoretical perspectives at various levels, is beyond the scope of this thesis. Future analysis based in this work may benefit from engaging with the sociological perspectives addressed above (amongst others) but this work restricts itself primarily to the micro level of analysis while seeking to point towards how this relates to potential analysis at the macro level within the same theoretical tradition.

2.7 Conclusion: an argument for the possibility of the coherent use of multiple theories

Therefore in conclusion of this review of a wide range of literature that is relevant to the research that constitutes the basis of this thesis, I argue that concepts of intersubjectivity based in Habermas' TCA can be coherently used alongside insights from other theoretical approaches. As has been mentioned earlier in this chapter, similar attempts have been made by the interactionist researchers, and the profusion of theoretical points of view in the analysis of mathematics education has led to a situation wherein the knowledge being brought to bear in research often reflects insights from multiple perspectives. However, as is made clear by the issues raised by Lerman, Bussi, and Ryve, this can be a problematic undertaking and it is therefore essential to be clear theoretically and methodologically as to how this attempt will be made.

Bussi (1994b) states that there is a need to 'look for' conceptual tools to deal with the use of multiple theories. This need for conceptual tools is based in the idea of complementarity of theories she develops as a solution to the diversity of theories in mathematics education. Yet what is the nature of these conceptual tools?

> It seems to me that the only solution is to accept *complementarity* as a necessary feature of theoretical and empirical research in the didactics of mathematics and look for conceptual tools to cope with it successfully, as Steiner (1985) suggests in the developmental program of the international study group on *Theory of Mathematics Education.* (ibid.)

Is there an approach which can be used to realise the conceptual tools necessary to achieve complementarity in the development of theoretical insights that make use of and are related to multiple theoretical perspectives? This is a question that will require further reflection. If one assumes that there is a way to develop such tools, then there is a need to identify what some of the features of these tools might be. One may be able to identify some of the characteristics of the conceptual tools by considering the ways in which authors have tackled the use of multiple theories.

Cobb (2006) raises two important points in relation to the use of multiple theories in mathematics education. First, in consideration of how various theoretical perspectives "…orient the types of questions asked and knowledge produced…" he suggests that the dichotomy between activity being viewed as primarily individual or primarily social in character fails to recognise the problematic nature of what is meant by the individual. Cobb suggests that, instead of positioning perspectives into these dichotomous categories, it makes more sense to compare and contrast the different characterizations and theoretical treatments of individuals. This is an important point with regard to the use of Habermasian theory as a large part of Habermas' project is a shift in focus concerning the treatment of rationality, away from philosophies of consciousness which consider rationality to be located primarily within the structures of the conscious subject, towards a treatment of rationality as inherent in the intersubjective features of communication.

Cobb's second point is that the use of multiple theories, and the relative validity of each, should be dealt with in a pragmatic fashion. This is not to say one should use whatever works in a non-reflective manner; rather it is based in Dewey's account of pragmatic justification such that a theory's ability to provide insight into empirical situations is a key factor in determining their truth.

> …the truth of fallible, potentially revisable ideas is justified primarily in terms of the insight and understanding they give into learning processes and the means of supporting their realization. (ibid.)

Using these ideas of complementarity and pragmatic justification one can see the development of models based in TCA (which address detailed analysis of evidence of intersubjectivity) as theoretical tools that can be designed in such a way that they may also allow for complementarity with other theory used in the analysis of the data. These tools will need to deal with the ideas of how the intersubjective understanding is conceptualised from the point of view of the TCA, and will seek their validity primarily in their pragmatic application of deepening insight and understanding towards the development of new courses of action that may be useful in achieving the common goals of the mathematics education community, such as they are. The aim here is to develop theory that provides insight in a manner that complements relevant theoretical work already existing in the field so that it may be useful to teachers and researchers. Failing this there is always the possibility of exploring in future work the potential networking of this theory and others (Radford 2008). This position is in line with the methodological arguments based in Habermas' philosophy, and focused on the rigorous use of multiple theoretical sources in the development of knowledge in the social sciences.

Chapter 3: Methodology and methods

> It is clear, then, that the idea of a fixed method, or of a fixed method of rationality, rests on too naïve a view of man and his social surroundings. To those who look at the rich material provided by history, and who are not intent on impoverishing it in order to please their lower instincts, their craving for intellectual security in the form of clarity, precision, 'objectivity', 'truth', it will become clear that there is only one principle that can be defended under all circumstances and in all stages of human development. It is the principle: anything goes.
>
> Feyerabend, P. 1975. Against method, New Left Books.

> A scientific approach is possible, but one must take care not to be scientistic—what counts are not the trappings of science, such as the experimental method, but the use of careful reasoning and standards of evidence, employing a wide variety of methods appropriate for the tasks at hand.
>
> Schoenfeld, A.H., 2000. Purposes and methods of research in mathematics education. Notices of the AMS, 47(6), pp.641–649.

3.1 Epistemological and ontological apologies

It is necessary to outline some thoughts on what I think reality is and how it can be known. The above quotes are interesting in that they span a certain space, from a radical (and perhaps mischievous) stance against the unity of scientific method from Feyerabend, to a more limited acknowledgement of the legitimacy of difference in approach and method from within the tradition of mathematics education by Schoenfeld. It is difficult for me to pin down exact concepts of science and social science and I have always been fascinated by such questions. Perhaps more so I have been fascinated by the fact that knowledge does progress despite the confusions and conflicts with regards to the security of foundations and definitions. In this section I will outline and discuss methodological positions that influenced my thinking during this research, but I find myself in the somewhat tenuous position of lacking certainty. Perhaps this is a virtue in light of Feyerabend's exhortation that one not succumb to one’s craving for intellectual security. However, there is a strong conviction behind the choice of the ideas discussed below as opposed to others that it is important to take a stand theoretically and ethically. The real ethical consequences of practical knowledge in the field of mathematics education demand a response that is equal to the challenge.

The research in this thesis serves as an exploration of the concepts of Habermasian critical theory in the analysis of small group interactions in mathematics classes. It is a technical analysis with potentially emancipatory consequences. It is also an exploration of the use of Habermas' approach to sociology from an intersubjective standpoint. The theoretical framework of Habermasian Critical Theory implies certain stances on questions of ontology and epistemology, on the nature of reality and our knowledge of it. I will briefly describe the weak realism that this position requires and which is articulated by Habermas. I will then outline Habermas' discussion on understanding meaning in the social sciences, which is a key methodological framework for this research. Finally I will discuss the particular research design and methods that I used in conducting this study. In this way I seek to elaborate the careful reasoning that serves as the basis for my research.

3.1.1 Realism and its limitations

What is realism? Habermas describes a weak realism with regards to ontology, one that attempts to preserve the socio-cultural realm of knowledge production while maintaining a link to a natural scientific understanding of the external world. This attempt to preserve the transcendental divide between externality and internality can be confusing at times but there are two sources that clarify some of the main points of this position. The first is Searle's discussion of realism (Searle 1997; 2010), and the associated articulation of the intertwined nature of objectivity and subjectivity with regards to epistemology and ontology. The second is a recently translated book on Truth and Justification, where Habermas revisits problems of epistemology and ontology (Delanty & Strydom 2003, p.460; Habermas & Fultner 2003). Below I briefly outline these aspects of the weak realist position and consider its potential limitations.

First let us consider Searle's discussion of knowledge and reality in his work on social theory (Searle 1997; 2010). There is a deep inter-relation between subjectivity and objectivity with regards to ontology and epistemology, claims Searle. Things can be ontologically subjective or objective and also epistemically objective and subjective. From this perspective it is possible to posit that one could have epistemically objective knowledge of ontologically subjective things (such as pain, or cognition). This conception of the relationship between subject and object is a feature of a realist position. Searle outlines a series of features of realism, which he takes as well defined and defensible. The key thing to note in this outline is that while it begins with the positing of an independent external world and ends with a claim to the epistemic objectivity of knowledge, in the middle it preserves a socio-cultural pluralism that serves as a foundation for objective knowledge.

> In light of the distinction between epistemic objectivity/subjectivity and ontological objectivity/subjectivity, we can identify the following structural features of our world view.
>
> 1. The world (or alternatively, reality or the universe) exists independently of our representations of it. This view I will call "external realism". I will refine its formulation later.
>
> 2. Human beings have a variety of interconnected ways of having access to and representing features of the world to themselves. These include perception, thought, language, beliefs, and desires as well as pictures, maps, diagrams, etc. Just to have a general term I will call these collectively "representations." A feature of representations so defined is that they all have intentionality, both intrinsic intentionality, as in beliefs and perceptions, and derived intentionality, as in maps and sentences.
>
> 3. Some of these representations, such as beliefs and statements, purport to be about and to represent how things are in reality. To the extent that they succeed or fail, they are said to be true or false, respectively. They are true if and only if they correspond to the facts in reality. This is (a version of) the correspondence theory of truth.
>
> 4. Systems of representations, such as vocabularies and conceptual schemes generally, are human creations, and to that extent arbitrary. It is possible to have any number of different systems of representations for representing the same reality. This thesis is called "conceptual relativity." Again, I will refine its formulation later.
>
> 5. Actual human efforts to get true representations of reality are influenced by all sorts of factors- cultural, economic, psychological, and so on. Complete epistemic objectivity is difficult, sometimes impossible, because actual investigations are always from a point of view, motivated by all sorts of personal factors, and within a certain cultural and historical context.
>
> 6. Having knowledge consists in having true representations for which we can give certain sorts of justification or evidence. Knowledge is thus by definition objective in the epistemic sense, because the criteria for knowledge are not arbitrary, and are impersonal.
>
> (Searle 1997, p. 150)

Habermas' position is thematically similar, defending a weak naturalism based in pragmaticism against a strict naturalism that is associated with the scientistic fallacies that he critiques in positivism.

> A 'strict' naturalistic explanatory strategy wants to replace the conceptual analysis of lifeworldy practices by a natural scientific one, for instance a neurological or bio-genetic explanation of the achievements of the human brain. A weak naturalism, by

> contrast, contents itself with the fundamental background assumption that the organic endowment and cultural way of life of *Homo sapiens* have a 'natural' origin and are basically accessible to an evolutionary explanation.
> (Habermas 2003, p460)

What are the limitations of this approach to reality and knowledge? Habermas and Searle are trying to salvage a rationalistic perspective on reality without succumbing to the totalizing themes of positivism. Do they succeed? They are trying to engage in a sort of detranscendental-ization, wherein the old tropes of dualism that have plagued Western thought for so long are abandoned, while at the same time the qualitative differences between the knowledge of the physical world and knowledge of the social are maintained. The danger from a critical point of view is that these attempts do not go far enough in rupturing the positivist reduction of the social to nature, and thus leave the ideological power of the status quo enshrined in a scientistic reification. Habermas, at least, seems confident that this is not the case, and that a weak naturalism can serve for a critical understanding of society. Yet the focus on rationality and the thematic search for universals is worrying, and hard to reconcile with the project to undermine totalizing social structures. It is clear that Habermas believes this is possible. In the discussion below, I touch on some of the methodological arguments that Habermas makes with regards to the researcher's potential to make use of communicative rationality to engage in critique. In general this position is appealing to me, despite my reservations. I have always believed in the external reality of people and things, while at the same time being unsatisfied with reductionist perspectives in psychology and social sciences. For the purposes of this study, as an exploration of the productivity of Habermas' ideas, I think that these conceptions of reality and knowledge will suffice.

3.1.2 Reflections on positionality

Originally I began with the idea that a methodological stance based in pragmatism and incorporating social constructivist, interactionist and socio-cultural perspectives supported the use of a case study approach to research design (Creswell 2003). I believed then and am still convinced that using a case study approach allows me to gain access to much of the complexity of interactions, discourse, competencies and knowledge in the classroom environment. Further this approach allows me to triangulate findings to support the validity of claims and insights derived from the study.

However in the course of the research I performed and its analysis, and in light of feedback from other researchers, I have realised that this is far from an unproblematic approach. The need to address with rigor the logic behind the use of multiple theoretical perspectives and the ways in which these overlap and depart from each other is a critical methodological issue if I am to support the position that these various theoretical and analytical positions can be used in a complementary fashion. What follows is a discussion of these issues that begins with my initial thoughts on the use of a pragmatic approach. This discussion leads into reflections on my adoption of a methodological stance of Critical Theory based in a Habermasian approach to the social sciences. This adoption of a Critical Theoretical stance was partly done in an attempt to address the multiplicity of theories at play in the analysis of the social both within the wider literature and in my use of such literature to inform analysis of the data in this case study.

Initial analysis of the critiques of pragmatism seemed to suggest that because of the emphasis that pragmatism puts on practice and action, it is often seen as naïve in terms of theory—that it has not dealt with the tacit theoretical aspects of its own program (Bishop et al. 2003). However, while this may be true in the common use of pragmatism, this is not entirely the case within the literature of pragmatism itself. Though the theory of the pragmatist may seem lacking in some sense, this is precisely because it moves away from the reified structures of modernity's entrenched mind-body dualism and begins to locate thought in the communicative problem-solving experiences of human beings. This can be seen in the traditions of Dewey and Peirce and Mead, as well as in the more recent uses of pragmatic theory in the social sciences as well as in its treatment by neo-pragmatists (Delanty 2005).

This movement is reflected also in critical theory with the concepts of the subject-object dialectic and the primacy of the object. This resonance between American pragmatism and the work of the Frankfurt school has been recognised by at least Adorno (in his lecture series 'An Introduction to Sociology') and by Habermas (in his explicit incorporation of pragmatic aims and ideas in his development of a Theory of Communicative Rationality)(Adorno 1999; Delanty 2005). Yet it must be acknowledged that critical theory and other approaches to the social sciences have not always meshed without conflict. It is of vital importance to address the reflective insights of the postmodern critique of modernity with regards to the central issue of subject matter in social sciences. It was my initial position, based on readings of Lyotard (1984), Foucault (1995, 2006), and others alongside my readings in Frankfurt school critical theory, that postmodernism is a vital source of reflective critique for modernity, but that by its very nature it is not an absolute negation of modernity but rather a critique which seeks to undermine the reified nature of totalizing conceptions. Therefore it should be possible to have a productive dialogue between the ideas of pragmatism and critical theory, and the critiques from postmodern social thought. And also in a similar way to have a productive dialogue between the multiple theoretical perspectives and their associated, though sometimes implicit, epistemological and methodological positions.

The position that I hold about the phenomena that I am studying, and my beliefs about the nature of social reality and the nature of possible knowledge about social reality, played an important role in the design and analysis of this research project. An argument can be made that the idea of 'researcher neutrality', in the social sciences at least, is in fact a passive ideological stance in support of the status quo, and passive only in so far as it remains unacknowledged as an ideological stance (Adorno 2000, p20).

This issue of researcher positionality has manifested itself in the social sciences in the form of a debate between a tradition of naïve and unreflective positivism in the social sciences (beginning with Comte and extending to the present day) and the radical relativism that some take to be the logical consequence of adhering to postmodernism as an epistemological foundation for knowledge of the social. In the context of these battle lines and the very real issues at stake with regards to rigor and professional integrity in research, it is reasonable and appropriate for me to acknowledge my own beliefs and positions so that I may at least communicate clearly the nature and scope of my claims to knowledge.

Initially, with regards to my own epistemological position, I was influenced by Frankfurt School Critical Theory, and saw research as work towards the unfinished project of modernity. Concerning the work at hand I was of a mindset that certain approaches to mathematics education are conducive to intellectual and therefore political emancipation and that such approaches could therefore be seen as goods in and of themselves. However, these positions do not negate the fact that knowledge of the social is needed in order to facilitate the implementation of such progressive practice and to understand the learning of students in these contexts.

As my studies progressed I worked on learning and using the methods of the social sciences in educational research. I continued to read and reflect on my own methodological positions both tacit and explicit. Two issues of central import to my thinking were the scope and limits of my own positions with regards to ontology and epistemology and the nature of the social: How might I be able to have knowledge of the social in a rigorous manner? And how might I understand and incorporate ideas and insights in the social sciences produced from positions that may not be identical with or which may even be logically incompatible with my own?

It remains difficult to articulate some of these positions with full rigor, and I find that when I engage in conversation with others about these topics I seem continually to realise new nuance, interconnections and points of departure. I think that these are some of the most interesting and confounding questions, and I will do my best to indicate the rough outline of my more or less consistent epistemological positions. First I am thoroughly a fallibilist. I am convinced of the contingency of knowledge and feel that the constraints, which exist on my experience of the world, are such that absolute certainty is continually elusive. That being said I am also of the position that I can know things about the world, physical and social, though clearly this knowledge will be contingent. These two fundamental positions seem to imply a need to rigorously delimit how and what I can know if I am to engage in the process of the production and dissemination of research knowledge. In the context of research it is my position that it is only proper and rigorous to take a stand with regards to these issues and defend it as best I can, noting of course that I am open to critique and debate on these issues.

Thus in the context of this research I have decided that a methodological position based in Habermas' ideas of Critical Theory will work well with my fundamental positions while at the same time taking into account the traditions of social science. It will also address the two main concerns outlined above, namely the need to delimit how I might gain insight from research produced with (both subtly and radically) different methodological positions from my own, and also the nature of the social and how I might go about gaining access to it. I will begin with an outline of some ideas that Habermas discusses with regard to the use of theory, and in the next section will discuss Habermas' position on the problem of understanding meaning in the social sciences in the context of decisions I made in the course of my research and their implications within a Habermasian Perspective.

3.1.3 Methodological decisions

The need to deal with the issue of theoretical compatibility of how different schemas can relate to one another methodologically and how they may be incommensurate is clearly an issue within Mathematics Education (Lerman 1996; Lerman 2000; Steffe & Thompson 2000; Lerman 2006). A methodological approach to this issue may be adapted from Habermas' position with regard to the plurality of theories in the social sciences. In addressing the relative claims and positions of psychology and economics, sociology, and political sciences in his work 'On the Logic of the Social Sciences' Habermas states, "…all three of these theoretical approaches can lay claim to a relative legitimacy."(Habermas 1990, p2) And that the 'negative inter-relation' of these approaches stems from, "…the fact that the apparatus of general theories can not be applied to society in the same way as to objectified natural processes." (ibid.) This leads to the assertion that,

> …the social sciences must bear the tension of divergent approaches under one roof, for in them the very practice of research compels reflection on the relationship between analytic and hermeneutic methodologies. (ibid.)

This relative legitimacy is a pluralistic pragmatic-epistemological approach and is related to ideas developed in Dewey and Peirce, two of the founding theorists of pragmatism. In the Theory of Communicative Action, this is also addressed when Habermas states,

> Critical social theory does not relate to established lines of research as a competitor; starting from its concept of the rise of modern societies, it attempts to explain the specific limitations and the relative rights of those approaches. (Habermas 1985a)

This approach can serve as a framework for examining the various theories and bodies of research that provide insight into the analysis of small group discourse in mathematics education. Situated theories of cognition, interactionist theories, anthropological theories, psychological theories, socio-cultural theories and linguistic theories may all be granted their 'particular legitimacy' while informing the overall analysis of the data in this study. It also allows for the potential to continually incorporate new theories of society and education and ask the question: 'What are the implications of one theory on the other? What are their relative rights and limitations?' Using this approach I shall try to respect the work that has been done already in the field of mathematics education to understand the types of interactions and other social data that are addressed in this research while maintaining a rigorous theoretical approach rooted in a methodological approach of Habermasian critical theory.

3.1.4 Belonging and the ability to participate in principle: necessities for understanding meaning in the social sciences

Early in my work on the research design, I decided that I should collaborate in some task design and instruction with the teachers that were the focus of my research. I felt that I had some good justification for it, but had to admit when pressed that I did not have a rigorous rationale, though it did fit with some of the ideas of participant observation. Based on reading and reflection on research design and methodology I decided to collaborate with the teachers participating in my research in the design of the tasks and lessons I was going to observe. I thought that this would help to alleviate some of the potentially negative authority issues of being a researcher working with teachers. Complementing this issue was that I was a very new researcher and recently had been a teacher, but only for 6 or so years. I certainly did not feel like an expert and I wanted to be authentic to my own knowledge and experience in my interactions with these teachers.

However the need for a rigorous rationale nagged at me, and I kept returning to it, as I felt that it was a central methodological decision that was crucial in understanding my own position with regards to research and practice. In the course of my research, reading and thinking I found Habermas' Theory of Communicative Action to be of central import in addressing this issue. This section will deal with his arguments addressing the Problem of Understanding Meaning in the Social Sciences (Habermas 1985a).

Habermas' ideas had come up again later in my research as I was in the process of collecting and analyzing some of the data from my research. In fact, as I was figuring out how exactly I could adapt parts of the theory of communicative action into an analytical coding scheme for student interaction, I ended up rereading large parts of The Theory of Communicative Action Volume I (TCA) (Habermas 1985a). It was during this reading that I realised that part of what had made me think that collaboration was important to me in terms of both method and methodology were claims being made by Habermas in a section entitled "Understanding Meaning in the Social Sciences". There was definitely a rigorous rationale behind these claims, if I could only get my head around it.

What follows is a brief description of Habermas' claims, some of their potential implications, and how they relate to my research design. In order to address the model for understanding meaning in the social sciences that Habermas has developed I need to address the idea of communicative action and some of the features that he ascribes to the concept. This begins with a definition of communicative action as coordinating action by achieving mutual understanding through consensus.

> ...the communicative model of action presupposes language as a medium of uncurtailed communication whereby the speakers and hearers, out of the context of their pre-interpreted lifeworld, refer simultaneously to things in the objective, social, and subjective worlds in order to negotiate common definitions of the situation. TCA, p95

This view is accompanied by a theory of implicit or explicit validity claims that accompany every utterance that correspond to the three worlds that Habermas refers to…

> ...an actor who is oriented to understanding in this sense must raise at least three validity claims with his utterance, namely:
> 1. That the statement made is true (or that the existential presuppositions of the propositional content mentioned are in fact satisfied);
> 2. That the speech act is right with respect to the normative context (or that normative context that it is supposed to satisfy is itself legitimate); and
> 3. that the manifest intention of the speaker is meant as it is expressed.
> TCA, p99

Or to put it more roughly the implicit validity claims in every utterance are: Truth, Rightness, and Sincerity. The explanation for how this works is that actors engaged in communicative action coordinate their action through the medium of language by reaching mutual understanding through consensus forming. Tacit validity claims accompany each utterance that refer to three different 'worlds': the objective world, the social world and the subjective world. At any point in the interaction any utterance can be challenged, which then leads to what Habermas refers to as 'Discourse'. The use of quotes in this situation is merely to note that Habermas' use of the term is significantly different from the way that other authors in the social sciences and in other philosophical traditions use it.

Discourse is technical as explained by Habermas, but is essentially described as attempts to re-establish consensus by addressing a validity challenge and justifying the particular validity claim that has caused the breakdown of consensus (and thus mutual understanding) or modifying the utterances and/or the claims so that consensus can be re-established. Having established this rough understanding of the framework of Habermas' Theory of Communicative Action, I now move on to reviewing how the implications of this theory play out in Habermas' conception of understanding meaning in the social sciences.

Habermas makes an argument that there is the potential for critical capacities inherent in communicative action. There are two major claims that Habermas makes in this argument. First that the social scientist has no special access to the lifeworld whose elements he wants to describe.

> ...In order to describe them, he must understand them; in order to understand them, he must be able in principle to participate in their production; and participation presupposes that one belongs. TCA, p108

And secondly that the fundamental structures of communication enable the social scientist to assume a critical stance with regards to understanding meaning in a particular lifeworld.

> The same structures that make it possible to reach an understanding also provide for the possibility of a reflective self-control of this process. It is this potential for critique built into communicative action itself that the social scientist, by entering into the contexts of everyday action as a virtual participant, can systematically exploit and bring into play outside these contexts and against their particularity. TCA p121

There are a series of questions that I considered in regard to Habermas' argument. They are as follows;

1: Why does the social scientist have to participate virtually in the interactions whose meaning he wants to understand? What is the significance of having to thus take a position on the validity claims that are associated with their utterances by the participants of that interaction?

2: What are the implications for researchers of Habermas' claim that the social scientist will only be able to link up his concepts with the conceptual framework found in the context of action in the ways that 'laymen' do in the context of everyday life?

3: How does this lead to the radical claim that these restrictions on understanding meaning actually furnish the social scientist with "…the critical means to penetrate a given context, burst it open from within, and to transcend it; the means, if needs be, to push beyond a de facto established consensus, to revise errors, correct misunderstandings, and the like."? TCA p120

4: How are we to understand the claim that the "same structures that make it possible to reach an understanding also provide for the possibility of reflective self control of this process. It is this potential for critique built in to communicative action itself that the social scientist, by entering into the contexts of everyday action as a virtual participant, can systematically exploit and bring into play outside these contexts and against their particularity."? TCA p121

The arguments that support these claims, particularly the last, are extensive and do address the questions above to a certain extent, though I would suggest that stylistically the argument can be somewhat vague and allusive in places. It is, however, extensive and is meant to be understood in the context of the wider theory of communicative action that Habermas is developing. I will not address it further beyond the questions raised here, but rather will reflect on some ways in which these ideas relate methodologically to my decision to make collaboration a focus of my research design.

As I considered the methodological relation of these ideas to my decision to use collaboration as an element of research design I began to think of it as securing a warrant between claims of access to the lifeworld, claims of being able to participate in principle, and evidence of participation. I will try to reconstruct some of my thinking along these lines in the next few paragraphs.

If one takes Habermas' claims to be valid, what are the implications in the context of my research? Reflecting on this there were two important implications that I thought to explore: First my own positionality and second the potential of this approach for critique that went beyond the particularity of the data. In order to understand the elements of lifeworld of mathematics teaching in England in mixed ability year seven groups, I needed to in some way belong to that lifeworld in order to be able to participate in principle in the communicative action I wished to research. This raised some issues with regards to my own positionality and also suggested some ways in which my decision to collaborate and participate may have helped with regards to securing access to the data (as well as the meaning of that data) that the research was focused on. While I have trained as a teacher and worked as a mathematics teacher in secondary schools for over 6 years, all of my professional experience had been in the US and specifically in diverse urban schools in the San Francisco Bay Area. There was a real risk that I did not belong to the community of mathematics teaching here in the UK in the same way as I did in the US. In particular I have found that there are all sorts of institutional, cultural, and attitudinal differences that make my background at least somewhat alien to those with whom I was working in my study. To what extent could I make the argument that I could participate in principle with the communicative action occurring in the mathematics classroom?

It may be the case that by collaborating with the participating teachers in the development of tasks and by participating in the instruction as a participant observer I have at least to some extent addressed with empirical evidence my ability to participate, and thus ability to participate also in principle (as this becomes crucial in the interpretation of data in whose production I did not participate directly). Thus my decision to make collaboration part of my research design may also serve to act as evidence that can be used to support a methodological claim to access to the data in the lifeworld on which my study was focused. While this in itself was an interesting methodological insight to me, the really exciting part of this was the potential for critique that Habermas identifies as a consequence of this approach to the social sciences. If I could properly understand the potential for critique that Habermas is trying to locate, then there was a potential to develop meaningful contributions to the field of mathematics education as a whole from the consideration and analysis of the data located in the particularity of my study. I kept this methodological consideration in mind as I moved forward with the analysis of the data.

This research is conceived of in the spirit of Habermas' theory of communicative action. It seeks to contribute to an exploration of the empirical usefulness of the formal pragmatic insights from the perspective of communicative action. In this discussion of Habermas and my methodological development I have indicated what I feel is the methodological potential for Habermas' ideas with regards to my research.

3.2 Research approaches and methods

A case study method, focused on the production of theories and models, which feature analytical generalization rather than statistical generalization, was used in this study. This decision was driven by the premise that a case study design would be able to focus on student learning in small groups in the context of complex instruction practices in mathematics classes. How could the study of a singularity be significant? I hoped to show that there is important technical content in the fleeting and ongoing interactions that occur within the classrooms. Understanding this in its particularity modelled to a certain degree how it is encountered by teachers and students. The focus of the study and what I hoped to learn from it was the main consideration with regard to the selection of a case study approach to research design. Further, this approach was in line with the methodological stances of weak realism and critical theory discussed previously. In the following section, I outline some of the ideas that influenced my research design, followed by a detailed description of the research design itself.

3.2.1 Two frameworks for case study: Yin and Bassey

The framework for this case study was adapted from elements of approaches to case study described by Michael Bassey and Robert Yin (Bassey 1999; Yin 2003). I began by using ideas solely from Yin, and it was only with the integration of Bassey's ideas that the design really came together. I continued to conceive of the design as incorporating elements of these two approaches but found that Bassey's ideas ended up being more productive in the design and implementation of this research.

Yin offers a technical definition of a case study as 'an empirical inquiry that 1) Investigates a contemporary phenomena within its real-life context, especially when 2) the boundaries between phenomena and context are not clearly evident.' (Yin 2003). His discussion of theory generation in the context of case studies is based in ideas from grounded theory, which has been the subject of much methodological debate over the decades since its development in Glaser and Strauss seminal work 'Discovery of Grounded Theory' (Glaser & Strauss 1967). Bassey addresses these issues and more in his review of scope and use of case study methods and methodology, where he discusses the history and developments of that approach from multiple perspectives including those of Yin (Bassey 1999). It is worth noting that recent developments in grounded theory have indicated an awareness and appreciation of the methodological developments in the social sciences since the approach was initially proposed (Corbin & Strauss 2008).

Bassey (1999) undertakes a reconstruction of case study in the context of educational research in his book, 'Case Study Research in Educational Settings'. This reconstruction takes into account the various work on case study research including the ideas of Yin described above. Bassey defines one category of case study as 'Theory Seeking' case studies. Bassey suggests that this is analogous with Yin's idea of the 'Exploratory' case study (ibid, p62). This type of case study is concerned with the exploration of a more general issue in the context of the examination of a particular singularity. This fits well with the aim of my research, which seeks to understand the technical features of learning in the context of the adoption of equitable teaching approaches.

Using these ideas I focused the research design on the process of connecting research questions to empirical findings. The model of case study research used begins with research questions, and proceeds through the collection and storing of raw data as data items, to an iterative process wherein the analysis of the data items generates analytical statements. These analytical statements are then tested against the data items and refined. Once the analytical statements have been refined as much as possible this iterative process is considered exhausted and the analytical statements are 're-expressed as empirical findings' (ibid). This leads to a useful idea that Bassey introduces in his reconstruction of the case study, namely 'Fuzzy Propositions' and 'Fuzzy Generalizations'.

Fuzzy propositions and fuzzy generalizations are set in contrast to scientific and statistical generalizations. They suggest that something may be the case, without attaching a measurement of its probability. Bassey suggests that such fuzzy generalizations and propositions are useful as a form of empirical finding in that they can illustrate the potential significance of research to other practitioners. This is especially important in case study research as it is already explicitly a study of a singularity that holds no necessary correspondence to the larger population of related social situations that could serve as a basis for statistical or scientific generalization. The research design for this study makes extensive use of generating analytical statements at the different stages of the analysis and testing them and refining them in the service of developing theories to understand the interactions, which are the focus of the research.

3.2.2 Data analysis: an integrated approach

The plan for data analysis was to adapt the constant comparative method as a rigorous way to address the iterative dialectical approach in Bassey's (1999) model for case study research. In Bassey's model there is a dynamic process of analysis between analytical statements and the data items. This was an important part of the analysis of data and the constant comparative method was put to good effect in the existing structure of the research design. I consulted Lincoln and Guba's 'Naturalistic Inquiry' as a place to find a discussion of how to extract the constant comparative method from grounded theory (Lincoln & Guba 1985). I also perused some of the other qualitative methods literature for discussions of the constant comparative method, for instance 'Collecting and Interpreting Qualitative Materials' (Denzin & Lincoln 2003). The integrated approach that I adopted combined the microanalysis techniques of open coding and constant comparison with the iterative analysis of Bassey's (1999) case study method.

Other sociological theories, as well as further use of Habermas, could prove useful in extending the analysis in this thesis to future work; however, this study confines itself predominantly to the micro-analysis of episodes of utterances in small group problem-solving, and thus is more focused on addressing a technical gap in the application of Habermasian critical theory to educational research. This gap in the application of Habermasian ideas is noted by Ongstad (2010), who claims that it is not merely an oversight, but a consequence of Habermas' neglect to explicitly explore the connection between the micro and the macro, in favour of philosophical macro analysis. While this may be true, it is also the case that Habermas lays out an extensive theoretical basis for exploring the micro level of utterances and builds a macro analysis of society on that basis. He also explicitly notes that such micro level analysis could be pursued, but he leaves it to others to do so while he pursues a theory of societal rationalization (Habermas 1985a, p.139). Thus the microanalysis developed in this study could potentially inform meso and macro analysis in the context of other sociological theories (see Chapter 9 Section 9.5) in future work. The development of this critical theoretical micro-analysis potentially addresses a gap in the literature, and could serve as a basis for future research that integrated more wide ranging analysis between different levels of discourses of power and ideology within mathematics education.

3.2.3 Salvaging the constant comparative method

The purpose of the constant comparative method in Grounded Theory is the generation of various categories, properties and hypotheses. It sets out to do this through a four-stage process in which one stage leads somewhat organically into the next. Beginning with coding data into as many categories as possible, the method then seeks to integrate the categories and their properties before delimiting and writing the theory that is being abstracted from the data. Where do the categories come from in the first instance? The constant comparative method suggests that while coding the data one compares each item to items in the same and different groups already coded in the same category. The constant comparison of the incidents soon begins to generate theoretical properties of the category. Yet how do categories emerge from coding items? Glaser and Strauss in their seminal book, Discovering Grounded Theory, suggest two types of categories that emerge: those that have been pre-constructed and brought with the researcher, and those that arise from the language of the research situation. There are some interesting claims about these different types of categories, for instance it is suggested that 'concepts abstracted from the language of the research situation will pertain to the processes that are to be explained and the concepts that come from the researcher are the explanations of these things' (Glaser & Strauss 1967).

The process of memoing during this constant comparative coding is key to the development of conceptual categories. Moving from mere comparisons of incidents with incidents- constant comparison leads to comparing new incidents with not only the incidents but also the conceptual and categorical features that have been abstracted from those incidents and their comparisons. Thus it seems that the categories and the relationships between categories are meant to emerge out of the reflective processing and interpretation that the researcher does in the course of analyzing the data with codes and analytical memos. Not only do data items pertaining to particular properties become more integrated, but also the properties themselves and their complex relationships to one another become more integrated. Finally, the relationships between the categories become clear in the analysis of the researcher and are integrated into a unifying theory (ibid.).

Then the process moves again somewhat organically into the delimitation phase where the Theory begins to 'solidify' and major changes become fewer. Further, a process of 'conceptual reduction' or condensation may take place wherein underlying unifying features are identified in the analysis of the data items such that the theory may potentially be constructed using a smaller set of higher-level concepts. Thus a process of constant comparisons of data items can guide the researcher to develop an efficient and more generalizable theory, according to Glaser and Straus (ibid.). At this stage the developing theory impacts the categories in that the researcher is concerned with the paring back of categories that originally emerged so as to focus more on those which pertain directly to the emerging theory. Finally, one begins to see theoretical saturation of the categories as new incidents may begin to contribute less to the emerging properties of categories and their relations. The idea of Theoretical Paring seems very important, yet it again raises the issue of where the theory comes from.

At the final stage of writing theory, the categories of the theory become the major themes in the writing and the primary support for these themes is the analytical memos. The coded data on the other hand serves as reference material for supporting the validity of various points, to identify strengths and weaknesses of the theory, and as a reservoir of illustrative examples. The goal of all this is to, as rigorously as possible, produce a theory which corresponds closely to the data. It seems that there is much that is worthwhile in the method of constant comparison, as long as methodological critiques of grounded theory are taken into account.

3.3 Research design

The case study focused on student interactions in the context of particular non-traditional mathematics pedagogy, complex instruction, in mixed-ability year seven classes. Three different un-set (or 'mixed ability') year seven mathematics classes at three different school sites took part in the case study research. Participants included 3 teachers, 3 teaching assistants, and 4 class groups of students. The initial contextual data included preliminary interviews of participating teachers and data from a summer professional development workshop on complex instruction, data from classroom observations and a professional development workshop done at the sites prior to the case study. This served as background information for the design of the case study and was also used in analysis in conjunction with other data. The plan was to collaborate in the design of curriculum with teachers and then observe student interaction in the context of the teachers' implementation of the lessons through classroom observations. These observations included some participant observation, and the use of video recordings of whole class and small groups interactions.

Table 2 Case Study Data

	Griffin Court	Summit Secondary	Green Valley
Planning	Interview/planning sessions audio x2 Lesson Plans and resources developed x2	Interview/planning sessions audio x2 Lesson Plans and resources developed x2	Interview/planning sessions audio x1 Lesson Plans and resources developed x1
Lessons	2 Lessons Flip video and whole class video (~12 Hours Video)	2 Lessons Flip video and whole class video (~12 Hours Video)	1 Lessons Flip video and whole class video (~6 Hours Video)
Reflective Interviews	Reflective Interviews x2	Reflective Interviews x2	Reflective Interviews x1
Contextual info; memos notes etc.	-Field notes -Reflective memos -Other professional development materials and recordings related to complex instruction used by participating teachers and schools		

The first stage of the case study, the collaborative design of conceptual curriculum with the participating teachers, occurred prior to the observations. The collaboration was within each school as the schemes of work already in place vary from site to site. This collaboration focused on design aspects of curriculum and instructional issues particular to complex instruction. The adaptation or development of group worthy tasks was supplemented by careful thought with regard to presentation and management of groupwork and whole class discussions. Careful thought was put into anticipating and planning for the opportunities and challenges that could be foreseen by the researcher and the participating teachers. This process was recorded through artefacts such as planning documents, curriculum documents, emails, audio recordings of collaborative development sessions and reflective researcher memos. This data would serve important situated analysis of classroom interactions.

The second stage of the case study consisted of video and participant observations of implementation of developed curriculum. The video sought to capture as much of the classroom activity as possible including student-to-student interactions, teacher to whole class interactions and teacher to individual and small group interactions. To capture this level of detail with video observation, one camcorder focused on the whole class, and six Flip cameras on group tables were used. Researcher memos supplemented video observations of the implementation of the curriculum. The observations attempted to capture teacher student interactions and student-to-student interactions. During the course of the case study, regular reflective interviews with teachers were documented through audio recordings. The interviews took place after every lesson observation. The focus of these interviews was classroom practice during the teaching of the planned activities. These interviews were unstructured and focused on documenting the perspectives and reflections the teachers had about the lessons observed.

The timing of the intervention and observations was staggered over a month and a half. I met with teachers to collaborate around the development of tasks and plans prior to the observations, on a time and day convenient to them with regards to their normal planning routines. I would then observe the lessons that were taught by the participating teachers using the tasks and plans developed. During these observations I made use of video cameras to record student interactions at each table as well as whole class interactions. After the lessons I met with teachers at their convenience to discuss the lesson. I repeated this twice with two of the teachers who participated in the study and once with the other.

3.3.1 Negotiating access and meaning

The research focused on the close examination of student interactions in classrooms where teachers were in the process of adopting complex instruction practices. This research was done with teachers who were also participating in the first stage of the research, which eventually led to the REALMS[14] research project. I was at the time a member of the research group that worked on this preliminary research and the REALMS project, which was headed by Professor Boaler (and during the second stage of research also Professor Judy Sebba). The sites and participants for the REALMS project were chosen from classes taught by a group of teachers that participated in a workshop on complex instruction held at the University of Sussex in the summer of 2008. Based on data collected at that workshop, the sites in the REALMS study were selected for their stated intention to eliminate setting in year seven, and to adopt complex instruction practices. Interested as I

[14] Raising Expectations and Achievement Levels for all Mathematics Students; http://www.sussex.ac.uk/circlets/projects/realms

was in delving deeper into student interactions in small groups in mathematics classes, I decided to take the opportunity to approach several teachers participating in the preliminary stages of the REALMS study to see if they would participate in additional research for my thesis. Three teachers at three different sites, who had been working on adopting aspects of complex instruction over the course of two terms, agreed to participate in the research for this thesis.

Many element of the research design required negotiation with the participants including: the collaborative curriculum development with the participating teachers and video observation of their use of the curriculum; the participant observation of classroom practice; and the reflective interviews conducted in conjunction with the video observation. I also engaged in some participant observation of lessons interacting with students in the course of instruction and in one instance in co-teaching a double lesson with one participating teacher. Participating in these ways- in conversations and collaboration around task and lesson development, and in participant observation as a co-instructor – was a series of decisions made in the context of negotiations of access with participating teachers and in light of methodological inclinations towards cooperative research practice. These decisions to participate had methodological implications with regards to establishing perspective as a participant observer as addressed in Section 3.1.4 of this chapter.

3.3.2 Arriving at a unit of analysis

In this section I give a brief overview of the journey from my initial ideas about my research questions and the associated unit of analysis to the research questions and unit of analysis that have become the central focuses of this work. This analysis also serves as an important contextual piece for understanding the situated nature of the cases. I then discuss the original ideas I had about a unit of analysis and how this changed as the focus of my research changed. I conclude by discussing the potential implications of this shift, how these have shaped my research and analysis, and how this shift relates to and in some ways deepens the initial vision of this research.

When I initially conceived this research my focus was on teacher and student action in the context of the adoption of complex instruction style pedagogies in mixed ability year 7 mathematics classes. My research questions were focused on how teachers and students dealt with emergent pedagogical factors (conceived of as constraints and affordances), what these emergent pedagogical factors were and the relationship between pedagogical approach, teacher practice and student learning. In the course of the preliminary stages of the research it became clear that in order to understand aspects of these questions it was important to examine what was happening in the student experience of the adoption of this pedagogical approach. The data focused primarily on teacher practice and reflection seemed to be missing key elements of the complexity that was evident from the observations of interactive problem-solving. Teachers were struggling, it seemed, to make sense of the complexity of the work being done by students when they engaged in this style of teaching. This struggle to understand showed them bringing into play notions of student learning, identity, assessment and feedback and a range of other conceptual tools.

Yet it seemed that key understandings of the form and content of interactive learning taking place were missing or underdeveloped. Many of the ideas that the teachers used were tacit or taken for granted. Others were partial or not fully articulated, for instance a professional sense that certain practices made sense without a secure understanding of why they made sense or how to build on them. One good example of the rather vague analytical assessment that some participating teachers employed was the notion of 'getting it'. They were very concerned with the students 'getting it', which is to say displaying understanding of the conceptual mathematics content at the core of a particular task or lesson. The teachers seemed to be confident in their ability to tell if students were 'getting it' from their informal assessment of students during small group encounters and the whole class plenary sessions, but this notion of what it meant to 'get it' and how that related to the complex interactions happening in the groups seemed important and it seemed to be something teachers were very concerned about and struggling to better understand. During my classroom observations I was struck by the complexity of the interactions taking place between students and between teachers and small groups of students. I was reminded of the complexity I had observed in small group interactions and had tried to manage when I had tried to teach in similar styles. I began to think that this was part of what was crucial in understanding the complexity of adopting complex instruction style approaches in mathematics and the associated challenges and opportunities. It was at this early stage that I refocused my research to try to make sense of student interactions.

After some thought and much reading on analysing small group interactions and discourse I arrived at the idea of 'episodes of utterances' as the unit of analysis (Bauersfeld 1995; Cobb et al. 1992; Cobb & Bauersfeld 1995; Habermas 1985a, p.101). This unit of analysis made sense given my initial analyses of the small group interactions and the developing conceptual categories focused on communication and evidence of intersubjectivity. Each utterance, in isolation, lacked meaning. Yet, by examining the role they played in episodes, it was possible to interpret their contribution to the collective construction of meaning as well as analyse challenges and obstacles to the development of construction of rigorous mathematical meaning. These episodes of utterances as a unit of analysis reside within the larger context, which serves to deepen the interpretive analysis in a situated fashion.

3.3.3 Transcription and analysis

Transcription of teacher interviews and whole class video and student interactions in small groups was done as data was collected, reviewed and reflected on in memos. This process took considerable time and was closely associated with the initial steps of analysis described below in the use of methods of open coding and constant comparison and the development of analytical categories that are discussed in detail in the next chapter. This transcription and coding allowed me to closely examine and reflect on the episodes of linguistic utterances in student interactions as well as the overall context of the classroom practice and discourse. Transcript are identified using a code that includes the date, anonymised identifiers of the school and teacher, the name of the task and which flip video camera the data is from. Thus a code might appear as '22062009GCMPFACTORSFP6': the first numbers represent the date of the observation; GC stands for Griffin Court (one of the school sites); MP stands for Ms. Phelps (one of the participating teachers); FACTORS indicates the task observed; and FP6 indicates which camera the data was from.

3.3.4 Challenges presented by the data

One of the most frustrating challenges of the data collected had to do with two tasks which seemed very productive in observations, but where the video and audio data were nearly unusable in the production of transcripts. In both the Counting Cogs task (Chapter 4 Section 4.3.2 and Appendix F) and the Crack the Code task (Chapter 4 Section 4.3.1 and Appendix E), the students were very active and engaged with the task during the observations. The plenaries in both class elicited interesting insights into the mathematics of the task, and as a participant observer I saw students engaged in interactive problem-solving. While this was very promising, the data from the recordings of these lessons had far too much background noise to be useful for transcription. Further, the video provided by the Flip Cameras was insufficient to interpret the students' interactions as it often only picked up partial movements and did not capture all the participants at any given time. Adding to this challenge both of these tasks focused on engaging the students interactively in ways that were not focused on primarily on whole group conversation. In the Counting Cogs task the students were manipulating papers cogs and exchanging cogs after investigating a pair and recording their findings. In the Crack the Code task the students were encoding messages in teams of two and then exchanging them with other students and trying to decipher each other's messages. In the first situation the students were very focused on the manipulation of cogs and what conversation was interpretable in the audio recordings was in reference to cogs and cog manipulation that was not really accessible in the Flip video data. In the second situation the decision to have students work in pairs in a task where they were keeping secrets from each other meant that the cameras had even less access to students interactions as the cameras were simply placed on the groups tables. Even when the data was more usable, the challenges of transcribing audio from noisy (if productive) groupwork classrooms was extremely challenging and time consuming.

3.3.5 Ethics

Research ethics in the social sciences is a complex subject. One way of understanding the complexity of this important aspect of research is to consider the 40 pages of the Economic and Social Research Council's (ESRC) Research Ethics Framework (REF). The document, intended to set out guidelines for 'sustaining and encouraging good ethical practice in the UK social science research', is framed by six key principles:

- Research should be designed, reviewed and undertaken to ensure integrity and quality
- Research staff and subjects must be informed fully about the purpose, methods and intended possible uses of the research, what their participation in the research entails and what risks, if any, are involved. Some variation is allowed in very specific and exceptional research contexts for which detailed guidance is provided in the policy Guidelines
- The confidentiality of information supplied by research subjects and the anonymity of respondents must be respected
- Research participants must participate in a voluntary way, free from any coercion
- Harm to research participants must be avoided

- The independence of research must be clear, and any conflicts of interest or partiality must be explicit

(ESRC 2005)

The REF notes that the responsibility of conducting research in line with these principles lies with the principal investigator, while institutions hold responsibility for 'appropriate ethical review, approval and monitoring'. While the research in this thesis was not ESRC funded, I used these principles to inform my consideration of ethics in my research proposal. The proposal was submitted in 2008 and approved in early 2009. I will briefly discuss some of the issues in relation to the principles listed above, as well as other reflections on ethics in educational research, in this section.

The most pressing concern to me as a researcher was to consider the potential harm that my research could do to participants. This is in line with my personal reflections on ethics over the course of my career. It is also in line with important new developments in ethics that are related to this thesis somewhat tangentially. As I already noted in Chapter 1 Section 1.1.3, there is the potential for the overall impact of education on society to be different than the intent of educators. This is related to a broad view of the nature of the unconscious in society and in individual action. Riker (1996) suggests that the discovery of the unconscious by Freud presents a serious challenge to classical notions of ethics: if one cannot be certain that the reasons we articulate consciously as being the motivation for acting are the only (or the true) reasons, then one is left in a position of potentially debilitating self-doubt (if one wishes to act ethically). Riker's response to this is to seek recourse in an 'Ethics of Health', suggesting that one must consider the actual physical and psychological health of others as a basis of guiding our action. While this argument raises serious theoretical questions about concepts of health (and in particular psychological health), it is an important acknowledgement that one cannot be content with having 'ethical reasons' for acting if our actions actually cause harm. While Riker's conception may not be the final answer to the age-old questions of ethics, it is an important pragmatic approach to dealing with ethics after the discovery of the unconscious. It points towards a local and particular understanding of ethical action that considers each case and each person and the potential harm that actions may cause in these local, particular cases. Reflections such as these have characterised my thinking on ethics throughout my career, and followed me from the classroom to academic research.

Reflections such as these merely accentuate the importance of community guidelines and processes of review and approval for research involving human subjects in the social sciences. I sought to address these ethical considerations on two levels in this research. First I sought to address the community standards by: recognizing the vulnerability of the population of participants I was working with; identifying potential risks; and seeking informed consent. The second level was to adopt a 'light touch' approach: the need for ongoing intervention and data collection was not indicated given the wealth of data collected in relatively short period of data collection, and the emphasis on qualitative analysis of particular episodes of utterances. From this perspective the fact that this research did not continue to amass data limits the potential unintended harm that it could have caused.

Researching students and teachers in schools is always problematic from the point of view of ethical guidelines. To what extent is it possible to seek informed consent, free from any coercion, from children (and their guardians) given the power relations that are inherent in the institutional relations of a school? To what extent is it possible to mitigate potential power relations between teachers and researchers? While these questions speak of absolutes, reality is a messy place where the best that can be done is to aim for the ideal in the context of the real.

With that in mind I sought permission from the teachers initially, and negotiated in good faith to design research that would not be a burden to their practice or conflict with their professional judgement. I sought permission from students and their guardians using a consent letter[15] (which the participating teachers introduced stressing the voluntary aspects while at the same time emphasising confidentiality of participation and normality of classroom practice during the research). Accommodations were made for students who did not obtain consent prior to the observations. These typically consisted of working with teaching assistants in a separate classroom for the lessons that were observed. Anonymity of all participants was respected by using pseudonyms, the video data of the classes observed has not been shared with anyone (outside some sharing at research meetings with other researchers on the REALMS project team), and the schools were made anonymous to the extent possible (the only identifying information is in the demographics which, by itself, is not enough to positively identify the schools). When I participated in the teaching in the classrooms I comported myself to the professional standards that I have upheld

[15] A sample consent letter can be found in Appendix A

throughout my career as a teacher and a researcher, which are based in a deep respect for others and reflections, like those above, that this consciousness of a respect for others by itself is not enough to ensure that my actions cause no harm.

The issues of power that arose in the analysis of small group interactions in this thesis research speak to the importance of this, fairly conservative, approach to ethics in social science research. The teacher, the members of the REALMS research team (including myself), and perhaps even many of the students, thought that the groupwork approaches that they were being used were likely positive and progressive, yet just beneath the surface in the black box of student group interactions was lurking unbeknownst to any of us the potential for conflict and power struggles.

3.4 Conclusion

This chapter has sought to bridge the gap between the abstract philosophical principles of methodology, the practical processes of case study research, and finally the precarious ground of ethics in social science research in school classrooms. This work serves as a basis, in conjunction with the articulation of researcher experience and research motivation in Chapter 1, and the review of literature in Chapter 2, for the analysis of small group interactions in unset year seven classes adopting complex instruction style practices. In the Chapter 4, I describe the classrooms, the teachers and look at the lessons that were observed as a way of situating the zooming in on episodes of utterances that takes place in Chapters 5 through 9. The structure of this thesis intends to communicate the rigor and the careful thought that went into the production of the knowledge claims articulated in Chapter 10.

Chapter 4: Situating the analysis of student interactions

> 'As subjects capable of speech and action, we "always already" find ourselves in a linguistically structured lifeworld.' (Habermas 2003, p.2)

> 'Contrary to the assumptions of mentalism, our cognitive ability can no longer be analysed independently of our linguistic ability and our ability to act, because as knowing subjects we are always already within the horizon of the practices of our lifeworld.' (Habermas 2003, p.30)

4.1 Setting the scene in the research schools

In this chapter I describe the interactions with teachers and students that took place in the course of this research. This description addresses the context in which the small group interactions, which are the focus of this research, take place and also highlights the aspects of the socio-cultural constellation that are not the primary focus of this analysis. These elements of the research narrative indicate the rough outlines of the lifeworld in which the objects of analysis are situated. This description of the setting, tasks, collaboration and participants leads to developing a rationale for focusing on small group interactions as a locus of important and hard to access practices, which are essential for participant understanding and learning. Finally, I reflect on the nature of communication with regards to utterances and semantically significant body movements, and how this can be addressed analytically. There are two key pieces here that are related to the quotes at the beginning of this chapter, first it is necessary to consider the lifeworld and background in which particular practices are situated in order to understand them, and second that the process of delineating a particular object of mathematics education research to analyse should also be a process of locating such an object in the context of a constellation of practices, agents, and objects that constitute the social reality of which the object of research is a part. These goals are addressed through the discussion of 'casing', the narrative description of the research participants and settings, and the discussion of units of analysis in this chapter.

4.2 A Case of communication in groupwork

> "For these reasons, consider cases not as empirical units or theoretical categories, but as the products of basic research operations. Specifically, making something into a case or "casing" it can bring operational closure to some problematic relationship between ideas and evidence, between theory and data. " (Ragin & Becker 1992, p.218)

What is this research a case study of? The strange contradiction of case studies is that they focus on the analysis of the particular, yet may seek to lay a foundation for claims generalizable beyond the particularities of the cases developed. The case is the bigger idea. It is a framing idea. Engaging in a process of 'casing' as discussed by Ragin, involves resolving problematic relationships between theory and data.

In this study, the process of coming to grips with what this was a case study of happened during the analysis of small group interactions from the flip video cameras. Originally approaching the data using a strategy of open coding, I quickly found theoretical ideas framing the way I was interpreting the small group interactions. This was the interesting piece. As a teacher I had been continually engaged in these interactions, working with one group then the next for over six years. These interactions seemed to provide a focal point for me as a practitioner. It was continually engaging and stimulating to be part of these endless conversations around mathematics problem-solving.

Part of what made it so engaging was that it was somewhat unclear how learning and teaching were developing, even when I was there speaking with and observing students as they worked in their groups. With 5 to 8 groups of four students in any given class I was in the constant position of not being privy to at least 80% of what was going on in the small group interactions at any given time. When I began this study, I set out to study constraints and affordances of classroom practice, in mixed ability, year seven, mathematics classes in England, but I soon came to realise that there was a critical technical domain that was not necessarily made clear with the situated approaches that I had begun with. I started to think of this domain as the 'black box' of student interactions. With the flip video data, suddenly I had access to hours and hours of data from this domain. Using open coding I began to dig into the small group transcripts trying to make meaning of the interactions. I coded each utterance, realizing as I did that the interpretations of the utterances only made sense in the context of the interpretation of the extended episodes of utterances.

Through the process of constant comparison and categorical refinement of the codes I was using I reached a point where it became obvious that my interpretations were heavily influenced by Habermas' Theory of Communicative Action. Specifically concerning the multiple validity dimensions of utterances and the way these served to allow for the collaborative construction of meaning. I went back and revised my research question to, "How can we understand student interactions in the context of small group problem-solving in mixed ability year 7 mathematics classes adopting elements of complex instruction?"

This was the process of 'casing' that Ragin describes, '...a product of basic research operations...', and the problematic relationship, between theory and data, was precisely how to understand these interactions in a more or less technical fashion in all of their complexity. This research became a Case Study of 'communication in small groupwork' in the context of mixed ability year seven mathematics classes adopting elements of complex instruction in England. The heart of the matter was the communication taking place and how to understand it. These phenomena were bounded by the context of the particular classrooms, schools and teachers and all of the practices and processes associated with these particulars. I began to see episodes of utterances, which constituted this communication, as my primary unit of analysis (although the utterance itself is also a necessary and interdependent unit of analysis). The rest of the chapter deals with the contextual pieces that form the boundaries of the case.

4.3 Working in three English schools

> "Studying intersubjectivity requires examining the resources, through language, that the teacher, texts, peers and others supply as well as the ideas that emerge in joint activity." (Lerman 2001, p.102)

During the course of this research I worked with three teachers and four different groups of students at three schools. Below I give a school-by-school account of the research that I engaged in: what was done; how; where; and with whom. All the events of this study occurred in May, June, and July of 2009. The intersubjectivity that this research focuses on, namely that which can be evidenced through communication in small group problem-solving, is located in the context of the collaboration with participating teachers, development of tasks, classroom practices, and small group interactions described below.

4.3.1 Summit Secondary School

Summit Secondary School is a specialist science college consisting of years 7 through 11, with approximately 240 students in each year group. The school is in the 14th percentile[16] with regards to deprivation in the community, and has Ofsted marks that are evenly split between good and outstanding. Two teachers from Summit Secondary School attended the Complex Instruction Workshop held at the University of Sussex in the summer of 2008, and they indicated their intention to unset their year seven classes and use elements of complex instruction beginning in year seven. When approached to be part of the initial phase of the REALMS project, they were eager to collaborate and invited members of the team, myself included, to come to their School. During the course of working with the teachers at Summit Secondary School, I approached one year seven mathematics teacher, Ms. Somerfield, to see if she would be interested in participating in my doctoral research in addition to participating in the REALMS project and she agreed.

Ms. Somerfield is a mathematics teacher who was in her third year full-time teaching at the time of the case study. She had been exposed to ideas about collaborative teaching and problem-solving approaches in her own secondary education, in her work as a teaching assistant, and in PGCE course and placements and was already experimenting with them in her classes. She participated in several professional development workshops around the use of complex instruction that were held at her school and was also engaged with local professional development groups that were focused on the use of small group problem-solving approaches.

Ms. Somerfield and I co-planned two lessons together, one was an adaptation of a task on comparing and interpreting graphs of track and field events (races). The second was a task using simple ‘codes’ to introduce ideas about functions and inverses. Ms. Somerfield led both classes and I took part as an 'extra' helper, in something like a Teaching Assistant (TA) role. There was another TA in the class on both occasions. Ms. Somerfield began the

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[16] Note on deprivation percentile: 1st percentile is least deprived and 100th is most deprived.

lesson on Olympic Graphs[17] by showing the students a video of Usain Bolt running the hundred meter dash and the 4x100 meter relay, and used this as a segue into a whole-class discussion of graphs of time and distance, and then introduced the task. The students worked in groups to interpret four graphs given to their group. Each student was to first interpret their own graph as though they were a reporter commenting on the race (this was done with reference to the commentary they had just seen of Usain Bolt's races). Then the students were to compare their graphs and determine which graph went with which event (100 meter dash, relay, etc.) and to record their reasoning and justification for their interpretation of the graphs. Before setting them off to work, Ms. Somerfield reminded them of the group roles they were using and also went over the 'success criteria' rubric that she had designed to help communicate expectations and frame formative feedback. The lesson was recorded with flip video cameras on each table. These were somewhat distracting to the students, but for the most part they left them alone or used them to record the work they were doing. After the lesson, Ms. Somerfield generously made time for a debriefing interview, which was audio recorded.

The second lesson co-planned with Ms. Somerfield was focused on introducing the students to the idea of functions and inverse functions. We focused again on how to design the task in order to make it 'group-worthy', that is, to design it so that students would need to interact with one another in order to complete the task. Whereas with the Olympic graph task, the students had an individual role (analysing their own graph) and a group role (comparing the graphs to each other), this second task split the groups of four into teams of two and set them against each other in a cryptographic game[18]. Following an introduction to secret codes where letters are replaced by numbers and then the numbers are manipulated, the student dyads were given two simple linear expressions such as [2x+5; 4x+10], instructed to devise a message, encode it with one of the two rules and exchange the coded message and the two possible rules used with the other team in the group. Then the challenge was to decode the secret message using the information given. The lesson proceeded well after some initial confusion on the part of students. Students were somewhat challenged getting into the task, but once they had begun coming up with messages and encoding them, they quickly moved into the task of figuring out how to decode the messages they received from other groups. In order to do this they had to come

[17] See Appendix D for Olympic Graphs task
[18] See Appendix E for the Code Game task

up with the idea of inverting the functions given to them with the code and figuring out which rule had been used. Ms. Somerfield reflected on this during the debriefing interview:

> I think they found it quite difficult just to get into themselves. I think maybe- I dunno I'm always torn about whether to give them more direction with things like that or not. I think it would definitely help them... but then actually when you see – the sort of penny drop with them and they sort of start getting it- it was really really nice. The whole sort of buzz in the classroom changed and um, you know, they all started to go 'oh yeah! I know how to do it now! Look hey cool, look lets give this a go!' and it was really nice to see. You lose, like time, but actually they get quite a lot more out of it.

Ms. Somerfield articulated a concern about the balance of giving the students direction with the goal of letting the students struggle to interpret the problem and struggle with the problem-solving on their own. She made the decision initially as she circulated throughout the class to help the students with the interpretation of the problem and stopped the class at one point to direct students to the task checklist and reiterate the steps involved in the task. After this intervention, the students engaged more productively and quickly began producing codes and exchanging them with other teams. The majority of student teams then figured out how to decode the messages without being directly instructed in the method of inverting the rules. Ms. Somerfield was struck by this student success:

> And lots of them they started to – they were doing it themselves, they were sort of like 'oh should we sort of like, reverse it? Shall we reverse what we did?' and I [said] ' hmm oh that's a good idea'. [laughs] It was really really nice, and they started to notice lots of patterns and how all the letters were ten apart in the function was ten lots of...

Ms. Somerfield also reflected on the challenge of using open-ended problem-solving in the context of the curriculum plan for year seven. This illustrates the challenge of managing external institutional expectations alongside the desire to engage students in authentic problem-solving practices. She identified this task as a good introduction to the concept of functions, which would need to be formalised in further lessons, and this was a reflection that was echoed by other teachers in the research with other tasks. This probably reflects on the nature of the tasks developed as well as the institutional pressures of covering the content detailed in the National Curriculum. One thing that did impress Ms. Somerfield was the progress that students made in the context of the problem-solving task:

> I enjoyed the activity- I'm not sure where I would place it. I think it's a nice way to introduce the notion of function and then you can sort of formalise it and make sure they're using the right sort of notation and that sort of thing, but it did really get them going... and within a lesson they started looking at functions and inverse functions straight away which is pretty amazing.

The collaborative decisions made to try to design 'group-worthy' features into the task and lesson plan also came up in the debriefing interview. This was a recurring challenge in the work to use complex instruction ideas, when interpreted as being dependent on students needing to interact in order to complete the task. This necessity is predicated on the idea that if students do not need to interact in order to complete the task then often many are denied opportunities to participate and thus status differences (with regards to academic status) are exacerbated as some students 'rush off', while other students are 'left behind'. There are many different elements that can make a task ‘group-worthy’, and Ms. Somerfield reflects on the impact that the design decisions made in this task:

> It's definitely good that we break them down into two teams for each table, but I think there was still, um, I mean there's certain individuals I know so well in that class and I know that they’re not going to work with other people [laughter], um, there's the boy sitting at the front, um, I can just see him straight away, he's got all the bits in front of him and he's just working it out. I mean he's a really clever boy, but he loves to do it himself...and nothing has stopped him so far, so I don't know.

This quote suggests how gender issues can present themselves in these classroom contexts. These kinds of narratives of gender may be symptomatic of wider narratives of gender and ability in the school culture that position some students as more able (Solomon 2007). It is difficult however to address issues of gender in the analysis in this thesis in a systematic manner given the snapshot nature of the data. Thus I simply note the potential impact of cultural narratives of ability both in the self-identity formation of the boy that Ms. Somerfield is referring to, and suggest that this could be indicative of wider patterns of identity positioning in the local school mathematics culture, which are beyond the scope of this study.

> I mean there's sort of scope for them- there were some groups where one team had got it and the other hadn't, and, actually, you know, the boys were sort of asking me 'how do you do it? How do you do it?' - they had seen me talking to the girls, and then they [the girls] sort of explained it to them [the boys]. And when I looked back over, they [the boys] were writing their code out and they really understood it. So they'd obviously done well in explaining it to each other.

Again there is interesting analysis from the teacher that describes the dynamics of interaction in the classroom. Beside the ongoing gender narrative in this excerpt, the way in which Ms. Somerfield characterised the interaction of students in the groupwork phase of the codes lesson suggests how she thinks about the merit of complex instruction style groupwork. She speaks of the girl students teaching some boy students and observing that the boy students had 'really understood it'. Ms. Somerfield reflects also on technical features of classroom management and grouping strategies.

> I think that splitting them into teams [of two] was really good because they had to work together whereas if they had been in a whole group [of four] it would have been much easier for them to kind of sneak away without doing much.

While this is the sort of professional practical reflection that allows teachers to be successful in the adoption of different pedagogical approaches, it also indicates that the teacher is acting strategically to achieve pedagogical goals. Is strategic action the only kind of pedagogical action that teachers can bring to bear? Questions such as these are brought up again in Chapter 9 and analysed in the context of the theories and models developed in this thesis.

Ms. Somerfield also reflected on other aspects of the task design, including her development of 'success criteria' into a rubric style feedback sheet[19] and plans to potentially include specific focus on reflective and peer assessment using the rubric. She reflected on how she planed to build classroom culture features like the use of success criteria tool over the end of one school year of into the next

[19] Can be found attached to Codes Task in Appendix E

> I don't know, I think next year, like, if they know that we're going to be coming back to that, success criteria, at the end, then they'll be trying to sort of meet it a lot more.

This suggested a belief on the part of Ms. Sommerfield that it takes time to establish practices that constitute change in classroom culture and her intent to build on the complex instruction practices. She went on to reflect on the way in which incorporating a 'checklist' structure for the students into the task helped her to be aware of what she was expecting and how she might intervene to achieve pedagogical goals.

> It's just, it's finding a way of getting them to sort of take notice... But if they're there then at least as the teacher, planning that, you know what you're looking for, as you're going around and... you know sort of where to point them...

Ms. Sommerfield was articulate in recounting interactions with students during the groupwork part of the class. She reflected on the amount of guidance she had to give the students, and how she tried to get students to help each other as a way of deferring authority back onto students. She reflected on strategies she used to get students to present to each other and the insight that this gave her into their understanding.

> I think I had to give away quite a lot at certain points- um- but, I don't know, it was fairly easy with them working in teams, like I said, to be able to pass that onus on to the other team and say 'well you know your group doesn't understand this at the moment, can you help them out? Like just picking on an individual to explain it to everybody- I just really like doing that, it's really nice to hear how much they understand, because often when you do that they're like 'hmmm, yeah, I dunno how I'm doing it...' but they are doing it and they're getting it right, they just can't quite communicate that to everyone else. But yeah, I felt today, probably more than in other tasks, I've had to tell them a lot more- about how to do things, rather than rely on their prior knowledge, but then as an introductory task, or something, it's bound to be that way, and with the algebra, you know, the [unclear] be advanced, so figuring out how to think about it themselves can be difficult, so...

However, this being said, it was acknowledged that Ms. Somerfield had not given them hints about the inverse process and many of the groups had figured this out, which suggested a certain amount of independent-of-teacher collaborative problem-solving. Then Ms. Somerfield reflected on the challenges she saw in teaching all her students using the groupwork methods.

> I don't know, I really enjoy it- my concern at the moment is those children who are sort of still, I can still see them dodging, not dodging the work, but the ones who aren't keeping up with it- they're the ones I sort of worry about, the ones who are storming off on their own and doing it themselves, at least the rest of the group can continue with it without them and make sure they understand it, but it's the ones who are sitting there looking a little bit lost, and how to sort of help them...

There was a narrative of confidence that cropped up, which at first seems professional and supportive. However, it bears noting that this kind of narrative of 'confidence' has been implicated in other research as being a tacit discourse of positioning around ability (Solomon 2007).

> And they are often the ones who do get a lot more support, with math intervention classes, so it's not that they are shying away from it, it's just that actually their confidence isn't coping with, um, sort of sharing ideas with other people, so I don't know, I'm thinking at the moment about how to deal with that, but I'm not really coming up with any solutions [nervous? laughter].

This next quote deepens the analytical theme of 'dealing' with less able students as Ms. Somerfield reflected on her views of students that would be categorised as having lower ability. She discussed how their instruction would be differentiated in a 'standard lesson'. She also reflected on her interpretation of Complex Instruction as 'trying to remove those barriers' (of differentiation). However the next part of the statement seems to focus on merely exposing them to more difficult mathematics, rather than any belief that they may be able to learn without the accommodations of 'different shaped work'.

> They are the sort of children, that in a standard sort of lesson you would have different shaped work for them. They wouldn't necessarily be doing exactly the same as everybody else- um- And I dunno, I guess the mixed ability is trying to remove those barriers, and it is important that they see that sort of work and have a look at it know what it's about.

My interpretation of these remarks was that they reflect the challenges of teaching in heterogeneously grouped mathematics classes where students often have different constellations of background knowledge and different identities as learners of mathematics. The worrisome part of the narrative of confidence was that it seemed to be indicative of the seeming intransigence of the challenge of disrupting negative student identities and fostering higher attainment and greater participation in the learning of mathematics. As Ms. Somerfield notes:

> But then, if they don't have the confidence to attempt it, then what can we do as the teacher?

4.3.2 Griffin Court College

Griffin Court College is a specialist media and language school consisting of years 7 through 11 with approximately 120 students in each year group. The school is in the 25th percentile with regards to deprivation in the community, and has Ofsted marks that are universally outstanding. Two teachers from Griffin Court College attended the Complex Instruction Workshop held at University of Sussex in the Summer of 2008, and they indicated that they already used mixed ability in years seven and eight, used problem-solving focused curriculum, groupwork, and were interested in using elements of complex instruction beginning in their year seven classes. When approached to be part of the initial phase of the REALMS project, they were eager to collaborate and invited members of the team, including myself, to come to their school. During the course of working with the teachers at Griffin Court College, I approached one year seven mathematics teacher, Ms. Phelps, to see if she would be interested in participating in my thesis research in addition to participating in the REALMS project and she agreed.

The first task I co-planned with Ms. Phelps was that of investigating patterns in the factors different numbers have[20]. Our discussion was characterised by trying to determine how much structure to give the students. Ms. Phelps was very conscious of the time constraint and making the task productive within the time allotted. Alongside this concern was a focus that the students should be 'getting something' out of it. This focus was towards creating a concrete goal for student learning, although it was clear that Ms. Phelps was

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[20] The Divisor Counting task can be found in Appendix G

comfortable with a certain degree of flexibility, she definitely wanted to have certain key ideas to focus on communicating to the students.

We began by adapting a task I had identified after Ms. Phelps told me that the content was to be focused on prime numbers and factorization. This task was drawn from an Interactive Mathematics Program (IMP) Problem of the Week called 'Divisor Counting'. As we worked with this task it became clear that while the task was rich it was somewhat daunting in its openness. We worked on developing class materials to use with the task. These included a task card that had the problem statement and prompts on it, as well as the group roles. The group roles had not been used regularly during the course of the year, Ms. Phelps said (perhaps twice a term). We also developed a rough 'write up' work sheet to focus the students on recording their problem-solving and groupwork processes

During the course of the lesson there was some distraction due to the cameras (but not a huge amount, and in some groups it may have actually engaged more students in the group interactions). Students began to engage in the task without really paying much attention to the write-up sheet and the idea that they should restate the problem and record questions they had about the task and identify a strategy to pursue the goals. Noticing this, Ms. Phelps called for all the inclusion students (role) to come to the front of the class to talk to her. There she gave the students instructions that their groups should be focused on getting the blue sheet filled out by the end of the lesson. This had partial success; some groups began to focus more on reflecting on and recording their ideas and strategies. At least one group continued to work on the problem in the way that they had been prior to this intervention without writing anything at all on the blue sheets. Many of the groups came up with interesting questions and strategies. One of these had to do with what the 'different kinds of numbers' that the problem referred to were. Another was a strategy of finding all the factors for the numbers from one to 100 by filling them in by multiples, first doing all the even numbers with twos and then doing all the multiples of three and so on.

At the end of the lesson the groups were asked to reflect in their groups on how things had gone. Specifically they were asked to reflect on how well they worked in their groups. This was a decision by Ms. Phelps that was made in the moment in the class as she had planned originally to have each group share something interesting that they had done in their groupwork. Ms. Phelps expressed concern that the groups had not made enough progress for sharing to be helpful. She decided, partly based on her sense that the groups were not working well together, that the best use of the time was for the groups to discuss how to work together better and how to proceed with the task in the next lesson. Two instructional moves of interest were a decision to use roles to communicate with groups regarding the importance of the write up worksheet as a focus for problem-solving; and the decision to have groups reflect on groupwork behaviour and plans for progress on task rather than sharing ideas and work.

In the reflective interview after the lesson, we discussed the lesson and how she might change it. One concern Ms. Phelps had was that there were not enough points of entry into the task (little multi-dimensionality). Ms. Phelps came up with the idea that rectangles with integer area could get at the ideas about numbers of factors and their relation to different kinds of numbers (prime, square, cubic, composite, composite with squares and cubes etc.). We discussed this idea a bit and Ms. Phelps got more excited about it and convinced that it would allow more students to participate in the activity while getting at the central ideas about factorization and prime numbers. I suggested that it might need to be made clear that we wanted only unique rectangles but Ms. Phelps quickly pointed out that if we asked students to explicitly include rectangles that only differed by orientation that we would end up with the number of rectangles equalling the number of factors- which is where the interesting content was located for us (and hopefully soon the students). It certainly seemed like a good way to add multi-dimensionality to the problem and Ms. Phelps added that it would connect to work they had already been doing about area in previous classes. Ms. Phelps decided to adapt the lesson for another one of her classes and teach it on the Wednesday the same week, I asked if I could film it and she kindly agreed.

Other concerns raised by Ms. Phelps included the use (or lack thereof) of group roles in the lesson:

> I don't think they used their group roles in this task; I would have liked them to focus on the inclusion person; in tasks like that students that have special needs don't get included as much...

This led to a discussion of group and individual accountability and how to structure feedback and tasks to emphasis the collaborative nature of the tasks. Reflecting, on the question of how to design lessons and tasks so that students are accountable for their own and each other's engagement, Ms. Phelps said:

> Sometimes I say 'I'm going to choose someone in the group to share, so you all have to be ready to share' we do that quite a lot...the group accountability piece is almost there, but because there's no individual accountability it's almost alright to be excluded- and sometimes I do lots of motivation things, so groups that are working the best will get a reward or a merit or whatever it is.

When I raised the pedagogical strategies, sometimes used in Complex Instruction classrooms, where the teachers do not allow the groups to progress to the next task until the teacher is satisfied that everyone has some understanding of the solution arrived at collectively, Ms. Phelps was not sure if this was a positive strategy to pursue:

> What I don't want to happen is for the people who are working really hard but are struggling more, to then feel under pressure from the others, because you know [the others may be frustrated at being held back]

This reflection seems to show a concern to address the needs of different students. However, it is perhaps significant that the narrative, of some students being held back by others, is key in the consideration of strategies for addressing rates of participation. This suggests a significant problem in the conceptualization of mixed-ability teaching, in that the focus is still on the different 'abilities' of the students, and seems to imply that this is not a challenge to be overcome through the development of students, but rather an intrinsic quality of students that is resistant to change. Struggling students will always struggle; students that rush ahead and show proficiency with mathematical practices will always be prone to being held back in their development by students who have different backgrounds and different fluency with mathematical practices. Even in teachers who are advocates of mixed-ability teaching the dominant narrative of ability is influential in structuring how teachers see students and how they seek to meet students' needs as learners.

In the next lesson, Ms. Phelps had adapted the factorization lesson such that it was focused on rectangles as previously discussed, and had made adaptations to the resources she was going to provide the students. She had made the write-up sheet on a giant (A3) piece of blue paper and provided resources including graph paper, counters, and rulers. There was also a large blue chart for organizing their data (though this was merely provided next to the other resources and not given out with the task and the write-up sheet). She gave a fairly structured introduction to the lesson that was preceded by a "no-camera" talk about cameras and how she expected students to interact with them, as there was some consensus that the students had been too distracted by them in previous classes. The introduction seemed clear and she spent about 15 minutes setting up the task and being explicit about what the student were to investigate. Two main points she made were: 1) that students were to investigate only rectangles with integer side lengths; and 2) that she wanted them to count rectangles that differed only in orientation as distinct (thus a 3x1 and 1x3 would both be counted). There was an interesting question regarding whether squares were rectangles, which Ms. Phelps turned back to other members of the class before stating clearly that squares were a special type of rectangle where the side lengths were equal.

The students began working and it was difficult to see how they were progressing. I wandered around the room observing and interacting with students about their work, asking questions and making comments and suggestions. I tried not to get in the way of Ms. Phelps or the Teaching Assistant. There were some students and groups who were more engaged than others, but all groups were making some sort of progress on the task. There were at least a couple times when I tried to interact with groups using the group roles to communicate norms and expectations without pretending to authority, which I really did not have in the class. Reflecting on this I realised that I was already assuming a position of authority in multiple ways by participating in a similar manner to the teacher and teaching assistant. By discussing the roles and asking questions about what their responsibilities were and what they were doing I felt I was able to make positive contributions to how some groups worked together in a relatively non-confrontational manner.

One group in particular that I spent time with was a group with a boy who was not very engaged and three girls who were more engaged (or at least two were very engaged, the third somewhat less so). The boy was not doing much and not really following the talk between the girls, who were exploring some rather interesting predictions about patterns and the connection between the rectangles and the number of factors that a number had. I tried to engage the boy and get him involved in the conversation that the other group members were having. I got him to do some work with markers (little round plastic chips), building representations of rectangles with them. This seemed to be productive and went along for a while.

Meanwhile I was challenging the other group members to consider how they knew some things they seemed to be taking for granted, such as the fact that they had found all the possible rectangles for each area. One girl explained that they knew they had all the rectangles based on an understanding of factors, and a very curious thing happened. I asked the boy if he understood the idea the girl had just articulated, and the girl in question said something to the effect of, 'He wouldn't understand it anyway'. This seemed to be a very charged and presumptuous statement. I replied with something along the lines of, 'Well that's not a terribly helpful thing to say' and tried to get the students to engage in discussing the idea with each other. Things seemed to work well, the boy engaged a bit more with making representations using the markers and the girls made more of an effort to work with the boy. Reviewing the video data later it seemed that the boy did struggle to participate but was more engaged (or at least went through the motions) for the rest of the lesson. Close analysis of the small group transcript in Chapter 8, Section 8.1, reveals that the group dynamics of this episode were in fact rife with issues of power and positioning on the part of several participants, including the boy whose ability was repeatedly denigrated.

What is particularly interesting about this is that it relates to something Ms. Phelps said as we were discussing the class after the lesson. In that discussion she referred to the group and suggested that part of the challenge was that the girls were spending too much of their time trying to explain things to the boy even though they had other ideas and could have made more progress with them. I mentioned the comment that one of the girls had made, and Ms. Phelps was shocked, though she herself had expressed frustration towards the boy's behaviour and lack of participation, as well as seemingly implying that he was responsible for holding the group back, and in light of the analysis in Chapter 8 this is not surprising. I asked Ms. Phelps if the boy had any recognised learning disabilities and she said that no, it was mainly behaviour with him.

This example of the way in which different needs and backgrounds of students can present challenges to the use of mixed ability groupwork begins to indicate some of the overlapping challenges of teaching mathematics using groupwork. The cognitive demands of learning and teaching mathematics exist simultaneously in the same space as the power and positioning demands of identity formation and maintenance. The boy is acting out. The boy may have learning disabilities, and certainly does not have the fluency with spoken and kinaesthetic mathematical practices that the girls in the group have and it seems probable that the boy is positioned as having low ability and low confidence in mathematics.

During the plenary Ms. Phelps had the groups share interesting ideas that they had come up with and had some idea of what she wanted each group to talk about. Things began to get complicated rather quickly. First, Ms. Phelps brought up the issue of some areas only having three rectangles and asked students to give some areas that only made three rectangles. She called on one boy whom she seemed to know had some ideas and asked him to give an example of an area with only 3 rectangles. He responded by saying 'Prime numbers!', which was where Ms. Phelps was going with that eventually, though it was not a correct answer to the question she had posed, and it kind of threw her a bit as she had been confident that the boy in question had seen the pattern that squares of primes have three rectangles while prime numbers had only two rectangles. Ms. Phelps reflected on her handling of this situation as being not ideal from her point of view.

Another example related to this occurred in the same plenary when one group shared a prediction/conjecture that was not generalizable beyond the (very limited) number of areas they had investigated. Ms. Phelps tried to handle this by critiquing it and suggesting that another group had had a similar idea but found out that it was not correct (in fact she had helped them to this realization at some point during the class). However, in trying to prompt the other group to explain why the first group's idea was not correct, the student in the second group started to talk about something else (what they had been going to share as a group- something slightly different) and things got a bit confused. Ms. Phelps reflected very astutely on these aspects of her practice and suggested that she would like to have handled the situation in a different way, going so far as to characterise her practice as 'bad' and 'not what I believe is right' in these particular cases. However, in the professional opinion of the author, overall the class was at least competently taught, the task competently designed (especially with the adaptations that Ms. Phelps had added since the first trial), and the students came up with many interesting ideas that they were able to articulate and share.

We discussed some other things regarding ideas about navigating and facilitating whole group discussions and I had some thoughts about how Ms. Phelps would be interested in working with the idea of building a 'taken as shared' body of knowledge with her class. Finally, it was noteworthy that the group with the boy and girls discussed earlier where there was conflict around ability and behaviour worked productively for part of the lesson and during the wrap-up one of the girls shared a particularly insightful comment about the reason that the number of rectangles was the same as the number of factors (this was in line with the learning trajectory imagined by Ms. Phelps in that they recognised that Ms. Phelps' constraint about having them include rectangles that differed only in orientation meant that the two factors that got included in each rectangle got counted).

In the process of negotiating access to Ms. Phelps' classroom she expressed being uncomfortable with regards to being taped (audio or video) and she suggested that she was acutely conscious of the cameras and felt as though she was not acting naturally but thinking about what she was saying with the camera in mind. She suggested that I might teach a lesson in her class and then we could have conversations about it. While hesitant at first, I quickly decided that this would be a very interesting thing to do and would help in the spirit of collaboration as well as addressing some interesting methodological issues. I suggested a compromise where I would plan the lesson and run it by her for revision, and then team teach it with her such that I would introduce it and perhaps facilitate the plenary. She agreed and we set a date for Monday week.

The lesson plan was to deal with ideas of Greatest Common Divisor (GCD) and Least Common Multiples (LCM); the lesson was a two-part block with lunch in the middle, with the same group that got the first version of the investigating factors task. I took the lead on designing this task, as I was concerned that I was imposing too much on Ms. Phelps' time, as she had remarked in one of our debriefing interviews on the amount of time it took to adapt tasks and plan complex instruction style lessons. The task[21] that I came up with had two distinct, yet related parts. The first was a lesson on the Euclidean Algorithm for the derivation of the greatest common divisor of two integers. I introduced the idea, shared an animated resource on the interactive whiteboard that demonstrated the geometrical construction of the algorithm, and then gave them a task that showed the same technique arithmetically. The task then asked them to investigate several pairs of integers of their

[21] The Euclidean Algorithm task can be found in Appendix I

choosing, determine the GCD, and then make conjectures as to why this set of rules works. The final part of the task asked the students to reflect on their work and come up with a question that might help them understand the justification of the algorithm better. After my introduction, the students worked for approximately 30 minutes as the teachers/researcher/TA's circulated before coming together as a whole class for the plenary. Each group shared their conjectures and/or questions with the whole class. At the end of the lesson I made a point of not forcing resolution saying, "So this is still an open question: Why does this rule work? Keep thinking about it and see if you can come up with an explanation by next week."

The next part of the lesson was the Cogs Task[22], which I adapted from an NRICH resource on common factors[23]. This wasn't exactly in line with the goal of dealing with LCM or GCD, but it was a fun looking task and I ran it by Ms. Phelps and she agreed that it did fit in the curriculum, so we pressed ahead. The NRICH resource has animated cogs with different numbers of teeth that turn each other. The questions were: When a dot is placed on one tooth to identify it, will it touch every gap in the other gear or will it miss some? Which pairs of cogs will allow the tooth to touch every gap and which will not? Can you find any patterns and make any general conjectures about when different pairs will 'work'? (i.e. allow the tooth to touch every gap) and which kinds of pairs of cogs will result in skipping certain gaps every time? The students where given cut out paper cogs with numbers of teeth ranging from 3 to 12 as a group, and instructed to investigate pairs individually at first, record their findings as a group, and then exchange one cog with another group member and continue investigating cog pairs until all combinations had been exhausted. Then the students were asked to develop conjectures about which pairs worked and why.

The students set off to work on the task in their groups and the three adults circulated in the class observing and interacting with the small groups. There was a high level of engagement in the task, although it seemed to take the students a lot of time to begin conjecturing and some groups developed more insight than others. During the plenary, the students shared the kinds of patterns they had found, and remarked on the relationships between their ideas and the ideas of other groups. Several groups identified the key concept (that if the number of teeth on two cogs shared a factor that the pegs would skip teeth).

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[22] The Cogs task can be found in Appendix F
[23] http://nrich.mathematics.org/4771

Some interesting insights into how the different pairs skipped teeth in different ways were brought up and discussed. Ms. Phelps was happy with the way that the lessons had gone and seemed to appreciate the contributions I had made to the collaborative effort.

4.3.3 Green Valley Secondary School

Green Valley Secondary School is a specialist mathematics and technology college consisting of years 7 through 11, with approximately 100 students in each year group. The school is in the 95th percentile with regards to deprivation in the community, and has Ofsted marks that are satisfactory or good. Two teachers from Green Valley Secondary School attended the Complex Instruction Workshop held at Sussex University in the Summer of 2008, and they indicated that they were going to unset their year seven with support from their head of school, and were interested in using elements of complex instruction beginning in their year seven classes. When approached to be part of the initial phase of the REALMS project, they were eager to collaborate and invited members of the team, including myself, to come to their school. During the course of working with the teachers at Green Valley Secondary, I approached one year seven mathematics teacher, Ms. Boxer, to see if she would be interested in participating in my thesis research in addition to participating in the REALMS project and she agreed.

Ms. Boxer is a teacher with over ten years experience teaching secondary students, however she had only just recently become interested in the ideas of using mixed ability and problem-solving to teach students mathematics as opposed to setting and differentiation strategies. Influenced by exposure to Carol Dweck's (1999) ideas through a workshop she attended, and by the position of her head teacher (who was emphasising mixed-ability teaching throughout the school), Ms. Boxer adopted the practices with enthusiasm. Entering her classroom, the walls were covered with student work, and significantly the roles played by students were a prominent visual element in her classroom set-up. There was an organizational chart on the wall which had the names of the students on cards with Velcro backs that could be moved about with ease to form new groups and change roles. When the year seven students came into the room, they looked to see what groups they were in before going to their assigned tables. They were obviously comfortable with the organizational practices that Ms. Boxer had been adopting around the use of groupwork. Ms. Boxer began the class by making sure the students in each group knew what their roles were. At this point Ms. Boxer had been using elements of the complex instruction approach for over 6 months, and the students seemed comfortable working together once the lesson began.

The task, which was collaboratively developed by Ms. Boxer, my colleague Lori Altendorf, and I, was an investigation of π, that is, the ratio of the circumference of a circle to its diameter. The investigation was framed as a story problem involving a jeweller who made bracelets and rings that he presented in boxes such that the bracelets and rings touched the sides of the box. The challenge that was presented to the students was to see if the could find any patterns to help the jeweller determine how much silver he would need to make a ring or bracelet to fit any sized box. This was conceived as a very open question, and information for various different possible questions that the students might ask was printed on slips of paper, so that if they wanted to know about the dimensions of the bracelets or other 'realistic' features of the problem, Ms. Boxer could provide that information when the question arose. This idea was borrowed from another teacher who participated in REALMS project. We provided the students with a number of cut out solid circles, a handout with boxes that fit certain circles snugly, and that was it.

The design of this task was, in retrospect, problematic in several respects. For instance, though it had the trappings of being a realistic problem, the contrived nature of the challenge was such that it almost certainly could have come across as just another arbitrary school mathematics task. Surely the jeweller would simply make the ring or bracelet, and then get or make a box to fit the jewellery (a much more straightforward task), than make rings and bracelets to fit arbitrarily sized boxes? Somehow this didn't seem like such a big deal to us when we made the task. Despite these flaws in the task design, the students were mostly engaged and made an effort to figure out what we were asking them to explore. There were four adults (Ms. Boxer, 2 researchers, and a TA) working with the students in this lesson and the students made significant progress in investigating the dimensions of the different shapes and the patterns in the ratios of diameter to circumference in the space of the hour-long lesson. The different groups made presentations during the plenary and several groups reported the on how they had determined that the ratio between the circumference and the 'width of the box' was about 3. There were also some rich interactions that were captured on the flip video cameras at the small group level that were later used to analyse episodes of utterances and teacher-as-participant interventions (analysis of these interactions can be found in Chapters 7, 8, and 9).

4.4 A rationale for constructing the object of research as small group interactions

> From a sociocultural perspective an object of research on mathematics teaching and learning can be seen as a particular moment in the zoom of a lens. (Lerman 2001, p.87)

What are seen through the lens of this research are the various attempts at communication and evidence of intersubjective interrelation of students through communicative interaction. When I was teaching I had students work on group problem-solving tasks and circulate through the classroom, much like the teachers in this study. I remember listening to students talk about the tasks they were working on and trying to assess and intervene as appropriate. The assessment and intervention were deeply rooted in my own background as a student of mathematics and philosophy as well as my training as a teacher. The class would come alive with talk; some students would lean together over problems while others became distracted in one way or another. I moved from group to group assessing whether students were on task, what they were talking about and doing, deciding whether and how to intervene. There was a wealth of student activity at any given moment, and moving from group to group I caught glimpses of students' interactions and collaboration, and I participated in these communicative interactions.

These interactions form an essential element of the constitution of the mathematical identity of students, yet they remained under-analysed. How did the collaborative efforts of students lead to the development of deep mathematical understanding? What was mathematical understanding? How were mathematical understanding, knowledge, and practices related to the communicative resources that the students were bringing to bear? While these questions have been addressed by research in mathematics education, the research in this study aims to gain insights into learning and teaching by focusing on the small group level of analysis and aiming to interpret the ways in which students bring communicative practices, in the context of their backgrounds, to bear on understanding mathematics using a theoretical framework based in Habermasian Critical Theory.

The subjective is still opaque, what is analysed are the practices that students actually bring to bear to try to make sense of mathematics in small groups, primarily linguistic (though in a broad sense). If one takes seriously the idea that social factors (objects and practices) are constitutive of learning, how does this express itself in communication and collaborative problem-solving in the mathematics classroom? What is the basis in small group interaction for the constitution of mathematical practices, knowledge and identity? Focusing the zoom of the lens in this research down past the level of school and departmental culture, past the level of whole class interactions (whether dialogical or monological) and down onto the observable, and in-theory accessible through participatory observation, interactions of students and teachers engaged in small group interactions, until the issues of structure and agency become blurred in the rapid fire back and forth of the participants seeking to understand and communicate with one another around mathematical problems and problem-solving. Then take a snapshot.

What is happening? When one perceives understanding and learning as educators and researchers, what is it that is perceived? This focus may allow the development of insights that can inform the practice of teachers in the use of small group problem-solving pedagogies, it may also inform the development of curriculum and tasks that can be used productively in such settings. It is not so much finding new things that students and teachers are doing, but rather developing a technical appreciation for the complex work that participants are always already engaged in, as well as the challenges they face, as they enact the practices that constitute teaching and learning collaboratively.

While there are advantages to this level of focus, there are also disadvantages, things are missed or de-emphasised, and some interpretations are valued over others. In this study the focus on the particular comes at the cost of knowing the participants, the schools and the teachers better. It comes at the cost of examining the systematic conditions that constitute the constraints and affordances that teachers and students operate under. Keeping this in mind is important so that the micro-analysis can be meaningfully related to the macro analysis of the socio-cultural domains within which the interactions are situated. The use of the theory of communicative action may allow this, as Habermas has developed a critical sociological theory of the interdependence of lifeworld and system based in the rationality inherent in language and communication.

4.5 Final thoughts and transition

The research in this thesis consists of a case study of communication in small groupwork in mathematics classes. This conception of the case was arrived at through the basic operations of qualitative research in the social sciences- observation, participation, and analysis. The process of casing (discussed above in Section 4.2), is closely related to the development of the unit of analysis (as discussed in Chapter 3, Section 3.3.2) and the construction of the object of research as small group interactions (discussed in the previous Section 4.4). The three participating teachers and their students, while located in different schools and social contexts, share the important features of practice including the use of unset classes, the adoption of groupwork on challenging open mathematical tasks and conscious attempt to use ideas from complex instruction to address the challenges presented by these changes in practice. The issue of the 'black box' of student interactions is relevant in all these classes. The fog of teaching with groupwork obscures the complicated interplay of meaning making and identity formation occurring at the small group level. The ideas of 'casing' and 'zooming in' represent an effort to understand the meaning making of students and teachers-as-participants in these dynamic situations. In the next section there is an articulation of the first stages of the integrated plan for analysis. These first stages of analysis include analysis of the communicative validity claims of students (and teachers) in interaction based primarily in close reading and coding of transcripts but informed by participant analysis (i.e. as observer in classroom, as insider as teacher both in situ and previously, as observer of flip video). The methodological approach of this research seeks to leverage the sense-making potential of participant observation and the wealth of data from audio-visual recordings. From this position it makes little sense to suggest that the researcher can make sense of the meaning of the interactions without adopting a stance of a 'virtual participant' in the interactions- it is as though the researcher was an invisible teacher observing the interactions of the students and making sense of what they were saying to each other and how they were acting and interacting. This chapter acts as a narrative context seeking to establish a warrant for this approach to the analysis of episodes of utterances in small group interactions.

Chapter 5: Development of an intersubjective framework

The following chapter outlines the initial stage of analysis: describing and reflecting on the initial coding of the transcripts; the development of codes through the method of constant comparison; and the development of theoretical categories based on these codes. In this process linguistic features of the episodes of utterances became a prominent focus of analysis. These features include grammatical and syntactical features, such as whether an utterance was a statement or a question, as well as semantic and pragmatic features, which are focused on how the utterances were interpreted as meaningful. The final section discusses the further development of an intersubjective framework for the analysis of the small group interactions as well as other useful theoretical perspectives and their relation to one another in the analysis of the data.

The initial work of data analysis used open coding to examine the transcripts of small group interactions. The rationale behind this approach was to interpret the data without a specific theoretical viewpoint in order to see what interpretations grew out of researcher interaction with the data. This being said, it is important to note that researcher neutrality is a problematic concept and that the aim was to develop theoretical categories based in the data as well as the researcher's knowledge and experience. The initial categories were based on open coding and constant comparison, as well as reflections on appropriate disciplinary theory and literature. One risk of this approach is that the analytical argument may be circular. Whether this comes about consciously or otherwise, the question should be considered: Is the incorporation of the concepts of communicative action and intersubjectivity due to a 'true perception' of the inherent nature of the talk as represented by the data and described by the initial open coding and related categories? Or is it rather that the researcher's reading in the area of communicative action and other linguistic and social approaches has led to a predisposition to interpret these patterns in the data?

The interpretations in this chapter rely heavily on the concept of the social scientists ability to meaningfully interpret data using the stance of a 'virtual participant'. In order for this to be meaningful, the researcher must depend on her/his background knowledge in the lifeworldy practices of the social domain being researched. The argument has been made previously as to how this stance has been established by this researcher (see chapter 3 section 3.1.4 and chapter 4). However, it is worth noting that this interpretive stance is similar in important ways to the notion of 'gaze' discussed by Bernstein (2000).

> The dominant perspective within any transmission may be a function of the power relations among the teachers, or of pressure from groups of acquirers, or, particularly today, a function of indirect and direct external pressures of the market or the state itself. Thus a perspective becomes the principle of the recontextualisation which constructs the Horizontal Knowledge Structure to be acquired. And behind the perspective is a position in a relevant intellectual field/arena.
>
> At the level of the acquirer this invisible perspective, the principle of recontextualisation structuring the transmission is expected to become how the acquirer reads, evaluates and creates tests. A 'gaze' has to be acquired, that is a particular mode of recognising and realising what counts as an 'authentic' sociological reality. (Bernstein 2000, p164)

While this concept is heavily couched in Bernstein's sociological theory, it has resonance with aspects of the stance of the virtual participant outlined by Habermas and discussed in this thesis (Chapter 3 Section 3.1.4). The interesting thing to note is the way in which the dominant perspective, which directly influences the 'gaze' is a product of power relations to begin with. Thus power relations have shaped already what is interpreted as meaningful, and the act of interpretation by 'recognising and realising what counts' also seems to entail an exercise of power. This is an important insight that could inform further reflection on the methodological basis of the virtual participant stance for interpretation of communication as meaningful.

Further, the notion of 'mathematical gaze' is discussed by Dowling (1998) in similar theoretical terms but with reference particularly to the mathematizing nature of the gaze, which re-contextualises non-specialized practices under the rationality of the discipline of mathematics (Dowling 1998). Dowling draws on Bernstein's ideas (discussed above), and also Foucault, who addresses a concept of 'Gaze' in his treatment of the 'medical gaze' (Foucault 1973). Yet, it is important not to reductively assume that the authors use the term within the same tradition and with the same intended theoretical meaning (though they may). Further, this realisation of the potential role of power in shaping the interpretation of the data (in this particular sense), occurred relatively late in the analysis, as issues of power became prominent. The scope of the implications that this idea (of the role of power in the shaping of knowledge in this particular manner) may have with regards to the work in this thesis remains undeveloped in this thesis, though in the course of the analysis of this thesis it has become clear that the role of power in communicative interactions is an important topic for further study in which these ideas could be revisited and developed in future work.

Thus, beyond acknowledging this point here (that there is a strong possibility that the interpretive acts based in the background knowledge of the researcher are shaped by power and also may entail a use of power to shape that which is interpreted) and making the analysis as transparent as possible to the reader so that they may make their own judgements about how power may or may not have influenced the interpretations in this research, this question is left for future work (for instance on refining the methodological positions developed from Habermas' Theory of Communicative Action in this thesis).

My methodological position is such that I cannot make a claim to objective neutrality or privileged access to the truth of the research situation. As a fallibilist and in adopting a methodological stance based in Habermasian Critical Theory, I have a disposition towards interpreting talk as having intersubjective properties. Further it is my position that it is these properties that allow the possibility of meaningful interpretation of the data. It should be noted that the securing of a relatively discrete methodological stance was a process that developed alongside the interpretation of the data in an interdependent fashion. The codes and categories I was assigning and developing while interpreting the data were heavily influenced by Habermas' Theory of Communicative Action. Reflecting on this led to a greater focus on these theories as useful for interpretation and also for more clearly demarcating my methodological position. In this way the interpretive framework for analyzing the data and the development of the methodological position worked alongside one another and informed one another. While this does not secure against charges of circularity, this recursive approach aims towards the development of a coherent position based in the empirical data.

5.1 Open coding

Eighty different codes were developed in the initial phase of open coding of the small group transcripts. As many of these had similarities, I attempted to determine more general codes that subsumed some of the more similar codes within the original set. It became clear that I was identifying the utterances categorically, as I repeatedly used words such as 'statement' and 'question' and 'action'. However, some of these categories seemed to overlap at times and this became an important feature in the development of categories and a relational schema. There was a difference between statements and questions that seemed to be useful for interpretation. Other descriptors that emerged in the codes included 'Challenge', 'Justification', 'Validity', 'Orientation', 'Clarification', 'Demand', 'Request for Clarity', and 'Response'. A list of the codes developed in open coding can be found in Appendix B. In the next section I discuss how constant comparison was used to develop categories further and the relationships between categories.

5.2 Constant comparison

Using constant comparison I continued to develop new codes while at the same time using existing codes. As I coded each new utterance, I considered codes that had already been developed, as well as assigning new codes where necessary. In this fashion I continued generating new codes until I reached sufficient categorical saturation to proceed with the analysis of categorical similarities. The techniques used are widely recognised in the field of social science research (Corbin & Strauss 2008; Miles & Huberman 1994), and were used in conjunction with the integrated analytical strategy based in Bassey's (1999) work on case study in educational settings. From this perspective the analytical process can be seen as one of developing analytical statements (the initial codes) and refining them against the data through constant comparison to arrive at more refined analytical statements (the final codes). These analytical statements are then used as the basis of a schema for communicative utterances (Table 3) and an intersubjective model for small group interactions (Figure 1), which can again be considered as analytical statements that need to be tested against the data and refined. This final stage continues throughout Chapters 6, 7, 8, and 9.

5.3 Articulating the conceptual categories developed through open coding

Using techniques of open coding and constant comparison, I developed preliminary conceptual categories. While the categories seemed relatively stable at this initial stage and seemed to be reaching categorical saturation, I continued be sensitive to alternative interpretations. The character of the codes was to some extent a reflection of my reading of Habermas' 'Theory of Communicative Action' (TCA) (Habermas 1985b; Habermas 1985a). However, my experiences as a teacher and the broader reading I had done in the field of mathematics education also contributed to this analysis. Habermas' TCA was a useful reference for interpretation of the categories that had been developed to this point. In particular it helped to make sense of the complex combinations of validity claims and challenges, concepts of discourse, and the coordination of action. The use of TCA also presented opportunities for contributing to critical theories of mathematics education by exploring connections between concepts at the level of semiotic details of classroom interactions and wider social issues of justice and equity.

The following sections discuss the development of the conceptual categories and outline my thinking as I proceeded with the analysis of the data. The conceptual categories are: Action; Statement; Question; Teacher Intervention; and Response. Category descriptions include discussion of overlapping sub-categories: coordinating; problem-solving; and discursive. Although specific examples of the use of these codes are not given here, Chapter 6 has many examples of these codes and an example transcript with codes can be found in Appendix J. Examples are not provided in this chapter as the codes have interpretive meanings that depend on the context of an episode of utterances as well as the virtual participant stance of the researcher (as discussed in Chapter 3 Sections 3.3.2 and 3.1.4).

5.3.1 Action

This category was difficult to develop meaningfully, as all utterances and linguistic acts are a form of action. The category 'action', as distinct from other categories, might have made little sense without further reflection and refinement. In developing this category, I sorted the open codes into groups based on perceived conceptual similarity. I refined codes originally characterised as actions into three separate subcategories; Active, Passive, and Discursive.

The data coded as 'Active Action' were characterised by words or deeds that took control of the group interactions. I used this code when students redirected or took control of resources, delegated sub-tasks, or directed other students to do certain tasks. In reflecting on this category it seemed that the idea of leadership, or perhaps more problematically, 'natural leadership', may have been related to this category. This suggested that there might also be power relations or status issues with regards to data in this category. While at times the data in this category represented positive and productive contributions to collaborative work, there were also instances where this kind of action was less helpful. Reflections such as these were noted as potential themes for further analysis.

The data in the category of 'Passive Action' were characterised by words or deeds that were directed or guided by an external source, whether it was the teacher, another student or even task resources. Again this appeared to be a potentially productive category for analysis of status issues, which is a theme of central concern to Complex Instruction pedagogy. Again these actions may have represented either productive contributions, or controlling behaviour that got in the way of equitable collaboration, or some combination. These ideas were pursued through the iterative analytical strategy (Bassey 1999) as discussed in Chapter 3 Section 3.2.2 and the details of this analysis can be found in Chapter 6 through 9.

The data in the category of 'Discursive Action' were characterised by deeds or actions focused on justification and mutual understanding. Whether articulating one's own understanding or supporting a validity claim through computation or the application of a mathematical property, these data were focused on providing reasons and it is this focus which led me to use the term discursive in the naming of this (and other) categories. The concept of discourse that I employed in the articulation of this category is one that has certain features as outlined in Habermas' TCA, in which discourse is a technical form of reflective speech focused on reaching rationally motivated consensus (Habermas 1985a). However, given the reference in this thesis to bodies of theory that use the term discourse in alternate manners (such as socio-cultural theory), I shall refer to this technical kind of discourse as 'validity-discourse'.

Given that the meaning of a speech-act depends on its validity claims, when these claims are challenged or rejected in the context of an interaction, the supporting reasons implied by the speech act must be made explicit in an attempt to re-establish consensus. It must be noted that there are at least two aspects to Habermas' notion of validity-discourse. First, validity-discourse can relate to any of the three categories of validity claims, which entail (roughly) truth, normative rightness, and subjective truthfulness (honesty); second, there are implicit features of validity-discourse that are necessarily extant in the give and take of reasons. One analytical implication of this was that I might be able to examine communicative productivity or breakdown in these multiple validity dimensions. These sub-categories were notable both in their theoretical richness at this level of analysis and in their connections to other categories that were being developed through the techniques of open coding and constant comparison. A chart of these codes can be found in Appendix B.

5.3.2 Statement

The category 'Statement' was another broad category developed in the open coding phase of the analysis and used to help sort the data into conceptual categories. In breaking down the broader category and examining the perceived similarities internal to the data, useful categories and sub-categories were developed. The four sub-categories developed in the process of constant comparison under the broader category of Statement are: Problem Solving (previously Misc./Focused on task); Coordinating; Validity Claims: Identification and Support; and Validity Challenges.

The data in the category 'Problem Solving' were characterised by being mathematical statements that were not discursive (i.e., not explicitly related to validity claims) or coordinating. Thus this category initially seemed to consist of miscellaneous statements primarily focused on the mathematical content related to the task. However, in the process of constant comparison, it became clear that the category is actually one of problem-solving, albeit problem-solving without significant immanent conflict. My interpretation was that the validity claims that accompanied the speech-acts in this data were tacit. At this point I reasoned that pattern analysis of the data might reveal relationships between the 'statement' and 'validity-discourse' data, providing further insight (i.e., evidence of the breakdown of consensus of meaning leading to validity-discourse situations which are then either resolved or not).

The data in the category 'Coordinating' were characterised by being normative with respect to productive groupwork. There was at least one instance in this category that hinted at potentially problematic power/status issues- a combination of taking control and denigrating another student's contributions. This category was interpreted as being related to the 'Active' and 'Passive Action' categories although it was characterised by more explicitly normative content. Coordinating action is also central to TCA, as Habermas describes it as one of the principle aims of speech-acts (achieved through reaching mutual understanding and consensus).

The data in the category ‘Validity Claims: Identification and Support and Validity Challenges’ might have been collapsed into a single ‘Validity-Discourse’ category. The data were characterised by explicit statements identifying, supporting, and challenging validity claims and, in a recursive manner, claims about claims. They were developed as separate categories, but it seemed likely that they were in fact one category of Validity-Discourse statements. However, I did not collapse these categories as they might have proven useful distinctions in higher-level analysis when looking at patterns of speech acts and discourse. As has been previously articulated (Chapter 2 Section 2.1.2), validity claims are central to the pragmatic theory of meaning in speech acts outlined in Habermas’ TCA. Thus far it seemed that TCA had the potential to serve as a rich theoretical framework for the understanding of the patterns of interaction in small groupwork in this study. The codes in the broad category ‘Validity Claims’ can be found in appendix B.

5.3.3 Question

The category ‘Question’ had connections with other categories that can be seen through the similarity in the subcategories: Coordinating Questions; Problem-Solving Questions; and Validity-Discourse Questions. What makes a question a question? The data and codes in this category were interpreted as questions, and the sub-categories suggest the variety of roles these questions played in the episodes of utterances.

The data in the category ‘Coordinating Questions’ were characterised by a focus on normative aspects of the task, including questions about roles and questions focusing the group on goals and teacher input. Roles are a feature of Complex Instruction pedagogy and act as normative structures that communicate expectations and productive dispositions with regards to working in groups. Students used questions about roles to orient themselves and each other towards the task at hand. This seemed like it may be another rich category for iterative analysis of the transcript data (this is pursued in Chapter 7 Section 7.2). Without wanting to belabour the point, it should be noted that coordination of action is central to TCA.

The data in the category ‘Problem Solving Questions’ were characterised primarily as either probing or clarifying questions. Ideas about such questions that influenced this analysis are based in my training and work as a teacher and studies at the Masters degree level.

Rhetorical, strategic and loaded questions interpreted in this category indicated that questioning was a linguistic tool that was versatile and often used by students.

The data in the category 'Validity-Discourse Questions' were characterised by validity challenges and an orientation towards each other's understanding and also reflective orientation towards students' own understanding. Some data in this category were interpreted as the explicit request for reasons to support tacit claims. This is an example of how ideas from the TCA were already influencing the analysis of episodes of utterances in the course of open coding and constant comparison. The initial codes that were categorised as 'Question' can be found into appendix B.

5.3.4 Teacher intervention

The category 'Teacher Intervention' was broad, but proved useful in making the codes that are focused on the teacher's involvement in the small groupwork distinct from the codes focused primarily on student actions and interactions. It was useful to understand that the amount of teacher intervention was a small proportion of the utterances in each group's work, although the teachers were very active during the majority of the groupwork times: moving from group to group; listening to what students said; and observing what they were doing and how they were doing it. Implicit in much of the interventions that took place was a wealth of professional, disciplinary and cultural knowledge. I attempted to infer the challenges that the teachers faced and the skills they employed to meet them by examining the way in which they intervened in student interactions. One analytical strategy I considered would be to relate the analyses of complexity of student interactions and of the strategic and communicative actions of the teachers to one another so as to gain insight into the relationship between teacher practice and student learning experience. The three subcategories that comprise the Teacher intervention category are: Coordinating Teacher Interventions; Teacher Validity-Discourse Interventions: Modelling Problem Solving; and Authoritative Teacher Interventions.

Implicit and explicit statements and questions from the teacher that focused on directing the student behaviour and orientation towards completion of work characterised the data in the category 'Coordinating Teacher Intervention'. Much of the data in this category represented implicit direction of action. Whether this was an attempt to divert coordinating authority to the students or communicate such an expectation of independent coordination, or was a cultural norm that was simply understood as explicit direction (e.g. 'Why don't we all sit down?') was initially unclear and thus identified as a focus for further analysis. My interpretation of this data was that it represented strategic use of power in the classroom on the part of the teacher. It seemed to represent communication of normative expectations with regards to coordinating action, a key element of groupwork.

Questions and statements that engaged students in mathematical discourse characterised the data in the category 'Teacher Validity-Discourse Intervention: Modelling Problem Solving'. Essential modelling strategies interpreted from the data included: Asking probing and clarifying questions; challenging the validity of student statements in a communicative manner pertaining to validity-discourse (i.e., demanding justification) as opposed to authoritative or authoritarian manner; and asking leading questions to reveal overlooked categories.

Teachers acting as experts to secure the validity of mathematical truth characterised the data in the category 'Authoritative Teacher Intervention'. Actions such as a teacher's confirmation of a student's idea or calculation as correct or incorrect based only upon the teacher's own implicit mastery of the discipline fell into this category. This could have been a problematic category, although I also grouped data where a teacher diverted authority for mathematical content back to the students in this category, which was contradictory to the above articulation. This category was notable because it seemed that the teachers were attempting to strategically use their status and knowledge to direct the students' thinking and problem-solving collaboration. In this manner it seemed to be a productive category for understanding how teachers used their knowledge of mathematics as well as their pedagogical content knowledge to intervene strategically to facilitate learning. This raised questions about the teachers' ideas of anticipated learning trajectories and how they impacted the teachers' strategic interventions in the course of groupwork. This issue became a focus of pattern analysis to examine the interaction of authoritative interventions with collaborative work. Appendix B contains a list of the early examples of codes in this category.

5.3.5 Response

The category Response had very similar properties to the 'Question' category as previously discussed. The primary difference is that there were many more utterances coded and subtle differences between the first level codes. The sub-categories Coordinating, Problem solving and Validity-Discourse were characterised by their position in the episodes of utterances as direct responses to other utterances, often referring to them explicitly.

Coordinating responses were part of the back-and-forth interactions involved in determining the goals of the group in the context of the task. Although many of these did have features of validity challenges, they were not about propositional content related to the task. Some codes in this category were responses to challenges to legitimate participation, which is an aspect of validity in Habermas' TCA pertaining to Authenticity (or subjective honesty). These statements were deemed coordinating in that they were focused primarily on re-establishing tacit consensus of shared goals, a prerequisite for communication, and hence, collaboration. Many utterances in this category were made in response to strategic teacher interventions that attempted to shift the course of the groups' interactions. The character of the validity challenges in this category was normative rather than 'constative'. Thus in my initial coding they got their own category (as opposed to being contained in 'validity-discourse responses'. The fully developed intersubjective model of student interactions (see Chapter 10 Section 10.1 Figure 30) was adapted, in part due to the validity challenges in this category, to introduce a 'validity discourse' loop coming off of the coordination process, which is intended to signal negotiations around coordinating action and determining goals. This category overlapped with other subcategories while occupying a different grammatical and communicative position in the interactions. These interactions between codes were noted as potentially useful in making sense of the communication at a level of pattern analysis.

The category 'Problem Solving Responses' was characterised by utterances that were focused on engaging other participants in discussion of the mathematical properties of the task at hand. In this sense the responses were characterised by responding to others problem-solving questions or by agreeing with others' statements or building upon them by adding conjectures or elaborating mathematical properties. In one case a code put in this category consisted of validating another participant's conjecture using a mathematical property previously identified. This category was focused on collaborative mathematical work, yet it was interpreted as interdependent with many other aspects of the interactions. Moving forward with further iterations of analysis it was important to develop how these different aspects of the communication worked together to facilitate meaningful group understanding or the tasks.

The category 'Validity-Discourse Responses' was characterised by responses that were specifically focused on issues of validity, particularly validity challenges contained in others' utterances. In some cases these utterances supported or objected to other validity challenges, while in other cases they were raising the grounds that served to back claims that had been challenged by other participants. Some of these justifications were correct while others were incorrect, and it was important to see how these different issues played out in iterative cycles of analysis. In other cases the responses themselves were validity challenges of other statements. One code in this category had to do with a response to a clarification of a previous claim that had been challenged. In this case the participant acknowledged the clarification as sufficient and dropped the challenge. In many ways the codes in this category fit well into the emerging patterns of communicative action that can be understood with Habermas' TCA. The initial response codes can be found in Appendix B.

5.4 The intersubjective framework and its potential for analysis

The goal of this initial microanalysis was to develop a theoretical perspective that was connected closely to the data in the study. Specifically, I was trying to interpret the data using methods of grounded theory while attempting to avoid the positivistic assertion that the data could 'speak for itself' or that there was some direct access to a reality represented by the data. In this manner the interpretive strategy acted as an associative process, in which I attempted to work closely with the data and quickly develop interpretations based on personal and professional knowledge. I then reflected upon and refined the interpretations and considered the theoretical implications. The use of Habermas' ideas had two sources: first, a methodological inclination towards Frankfurt School Critical Theory; and second, the perception of accessible meaning constituted in the interactions between participants, and that interpretation of such could lead to key insights related to the research questions.

Ideas about intersubjectivity have come up in the literature of mathematics education in the interactionist perspectives developed in the 1990s, as well as in the socio-cultural and communicative discussions in the past two decades (see discussion in Chapter 2 Sections 2.2). The re-conceptualization of the subject that is required by theories of intersubjectivity has been raised in the exchanges between Lerman, Steffe and Thompson (Lerman 1996;

Lerman 2000; Steffe & Thompson 2000). The further development of theoretical perspectives dealing with intersubjectivity may be important as it may impact on issues of identity and agency prominent in mathematics education research (see discussion in Section 2.3). The use of Habermas' integrative social theory could provide an important theoretical bridge between the linguistic and semiotic research and the socio-cultural and identity research in mathematics education.

From a critical perspective, the nature of the subject is implicated in the aspects of western modernity that have lead to an age of 'Wonders and Horrors' as termed by D'Ambrosio and discussed by Ole Skovsmose (D'Ambrosio 1994; Alro & Skovsmose 2004). The connection between dualistic concepts of subject and object and the wider social impact of modernity is one that was a central focus of Frankfurt School Critical Theory (Adorno, 1999), the full extent of which is difficult to summarise. However, should such a link exists, this becomes a particular problem for mathematics education. Many of these 'wonders and horrors' have technical foundations in mathematics (Skovsmose 1994, 2005). The analysis of intersubjectivity in this thesis seeks to provide technical insights for teaching and learning from a perspective that is potentially not merely instrumental and which may be able to contribute to wider analysis in critical mathematics education.

The following section deals with theoretical integration of the codes and categories developed through open coding and constant comparison, and articulates initial theoretical perspectives for the analysis of intersubjectivity in mixed ability year seven mathematics classes.

5.4.1 Initial thoughts on integration

The process of using open coding and constant comparison raised a certain amount of ambiguity, which arose partly from the fact that initially there was not a clear theoretical framework being applied to the data. An attempt was being made to develop a theoretical framework that was responsive to the data and to the researcher's knowledge of the field of mathematics education and social science. These ambiguities were important to resolve to some extent in the formation of an adequate theoretical framework. However it was also important to consider the subtlety of the initial interpretations, and to address them without losing the original interpretive meaning of the analysis. This being said there was some categorical combination and elimination as the theoretical framework for interpreting small group interactions was developed.

Primary amongst these adjustments were the categories of action and statement. As stated in regards to the category of 'action', all utterances are forms of linguistic action. The theoretical resources that were decided upon in the course of this initial analysis were based in Habermas' Theory of Communicative Action. The category 'action' was used more initially and then less and less as the patterns of statement, question and response became clearer. Upon review it seemed that many of the codes in the category of 'action' could be reinterpreted into the categories of 'Coordinating Statements' and 'Problem Solving Statements'.

Another issue that needed to be addressed was the overlap in categories, especially in the sub-categorizations. In this context the issue of ambiguity was resolved by trying to conceptualise the interactions at two levels. On the first level was a grammatical/syntactical structure and on the other level was a level communicative function. Thus the broad categories initially included: Action; Statement; Question; Response; and Teacher Intervention. The overlapping sub-categories, meanwhile, were characterised by Validity, Problem Solving, and Coordinating utterances. This overlap suggests a potential schema for categorizing the utterances displayed in Table 3 below. Table 4 expands on the 'sub-codes' that appear in the overlapping categories in Table 3. This integration of codes into a broad framework contained implicit theoretical assumptions that are developed in the next section.

Table 3 Communicative Utterances

Communicative Utterances			
	Statements	Questions	Responses
Validity	SV, SVCh, TD, AD	QD, TD	RD
Problem Solving	SPS, AP	QPS	RPS
Coordinating	SC, TC, AA, AP	QC, TA	RC

Table 4 Code Key

Code Abbreviation	Code Name
SV	Validity statement
SVCh	Validity statement: Challenge
TD	Teacher modelling discourse/communication
AD	Discursive Action
QD	Discursive question
RD	Discursive response
SPS	Problem solving statement
AP	Problem solving action
QPS	Problem solving question
RPS	Problem Solving Response
SC	Coordinating statement
TC	Teacher coordinating statement
AA	Active Action
AP	Passive action
QC	Coordinating question
TA	Teacher action
RC	Coordinating response

5.4.2 Hypotheses on an intersubjective model of student interactions

The following analytical statements focus the further analysis of the data in the iterative manner outlined by Bassey (1999). These theories are based explicitly in the concepts of the TCA, marking a departure from the strategies of open coding and grounded theory. These strategies helped to clarify that theories of intersubjectivity based in the TCA were central to the interpretation of the data. Once this became clear it was important to focus on this and develop it using the iterative analytical strategies discussed previously (Chapter 3 Section 3.2.2) to further the development of a theoretical perspective and set of tools for use in the analysis of episodes of utterances in collaborative groups.

In year seven mixed-ability groupwork in classrooms adopting complex instruction style pedagogical strategies, small-group participant interactions around mathematical tasks can be characterised as having multiple grammatical elements, which address different aspects of communicative action. The broad grammatical categories are Statements, Questions, and Responses, while the communicative functions these can address are coordination of action, problem-solving (or 'constative'), and validity-discourse. Meaning is not entailed by the grammatical features of the utterances but rather in an interpretative fashion that is broadly pragmatic and based in the ideas of virtual participation discussed in Chapter 3 Section 3.1.4.

In order to understand how these grammatical and communicative categories relate to each other, I draw on Habermas' theory of communicative action. I outline the relevant particulars of the TCA and show how the aspects of my analysis relate to it and contribute to an understanding of how the elements interact. Habermas (1985a) identifies communicative action as the coordination of action through reaching mutual understanding based in consensus around a set of implicit validity claims. Communicative Action is a process of negotiation of common goals and the coordination of participant action to achieve those goals through achieving mutual understanding. The process of negotiation is in the context of the multiple validity realms or 'worlds', including truth, subjective honesty, and normative rightness. My interpretation of the grammatical features identified is that they are aspects of the common language that is presupposed in Habermas' theory of communicative action. I suggest that the categories of statement, question and response are structural aspects of the grammar of the common language shared by the participants. The separation of 'grammatical' elements from 'communicative' elements is somewhat artificial, and a consequence of the open coding from which these categories were developed. Grammatical, syntactical, and pragmalinguistic features of utterances may well be embedded and overlapping in episodes of utterances. While the interpretive moves made in this thesis are not the only possible ones, they lay claim to a local and contingent truth as per discussion in Chapter 3.

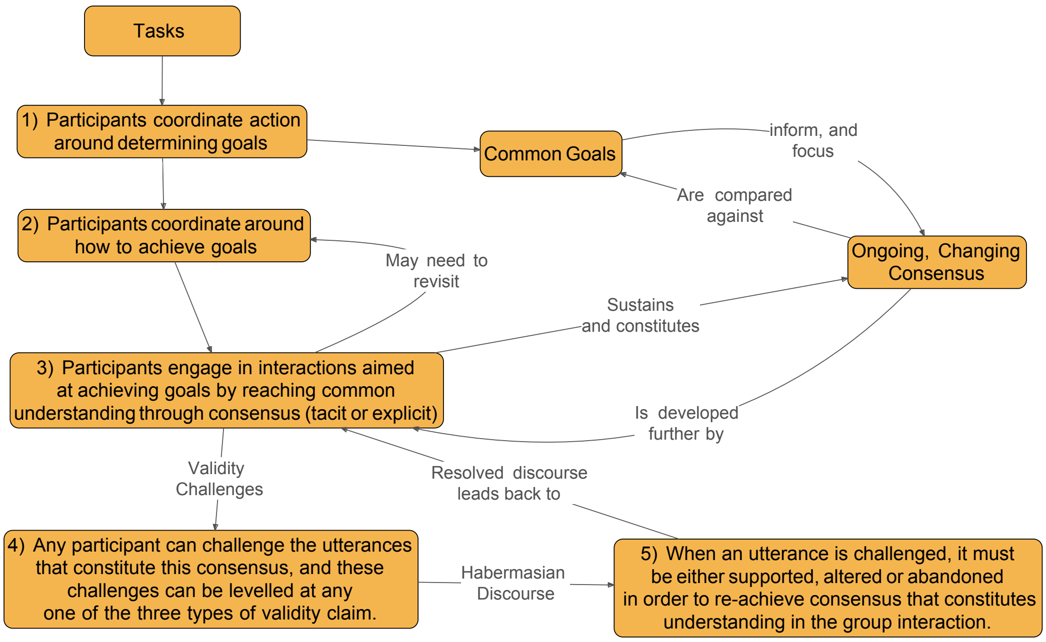
Two of the communicative categories address aspects of the Theory of Communicative Action, specifically the 'Coordinating' and 'Validity' categories. The third communicative category, 'Problem Solving', may also be considered as mapping onto the theory of communicative action in terms of the propositional form of the utterances about mathematical facts and practices. In the theory of communicative action, utterances are considered to have propositional content that refers to different realms. In the case of mathematical problem-solving, the realm is that of 'truth'. Problem-solving utterances that refer to the empirical features of a task, or a mathematical concept or practice, can be interpreted as belonging to this 'world' of 'truth'. While there are normative and/or subjective elements to these problem-solving utterances, there is an emphasis on a domain of mathematical objects and relations that are not clearly arbitrary subjective or normative constructs. The coordinating category contains utterances that emphasise the normative aspects of the interactions, and the validity category addresses utterances that are about challenges and justifications of the validity of propositions that may refer to any of the three realms.

I now turn to the ways in which communicative action takes place as a process in the TCA and use it as a model to relate the categories and codes to one another. This is done as the conceptualization of the codes implicitly suggests inter-relations. In the TCA Habermas (1985a) describes communicative action as a process of coordinating action by achieving mutual understanding of goals and means to achieve them. In this process of communication, utterances of the participants implicitly raise validity claims in each of the three realms (normative/ objective/ subjective). In the course of coordinating goals and related actions, other participants can challenge these claims and when this happens the participants enter into 'discourse' (Habermas 1985a, p42). This validity-discourse is one that is focused on explicitly re-establishing the tacit consensus that has broken down.

The model is a cyclical interactive process with a number of steps or stages:

1. Participants coordinate action around determining goals
2. Participants coordinate around how to achieve goals
3. Participants engage in interactions aimed at achieving goals by reaching common understanding through consensus (tacit or explicit)
4. Any participant can challenge the utterances that constitute this consensus, and these challenges can be levelled at any one of the three types of validity claim.
5. When an utterance is challenged, it must be supported, altered or abandoned in order to re-achieve the consensus that constitutes understanding in the group interaction.
6. The interaction cycles through steps 2 through 5 until the participants decide they have achieved their goal outlined in step 1.

Figure 1 Initial hypothesis on an intersubjective model for student interaction



There are two main elements that are not part of these steps but which are shown in Figure 1. The first is the tacit or explicit common goal, and the second is the ongoing, fragile and temporal consensus entailed in the problem-solving discussion. Also missing in this model are the relationships between consensus and action, task completion and understanding. If the task is well designed it is possible achieving the goals set out in the task entails the understanding of certain mathematical concepts and/or knowledge. Thus by coordinating their actions to achieve the task, the students may have intersubjectively understood the content. For instance, step three could be amended: '…and then do the actions that correspond with that consensus which in aggregate will achieve the common goals (in this case the product of the task)'.

It is important to note that the term discourse is used in a much narrower sense in Habermas' work (Habermas 1985a, p42). This does not imply that Habermas ignores the other broader use of the term referring to these ideas in terms of particular cultural settings and/or contexts. Analytically, Habermas considers all participants in a communicative interaction to have equal rights and opportunities to participate. This is due to the presumption of an ideal speech situation (see Chapter 2 Section 2.1). Communicatively, teachers have no recourse to authority except through their ability to bring to bear the 'unforced force of the better argument'. This is again a counter-factual idea conceived of abductively by Habermas as a prerequisite for communicative action.

5.5 Final thoughts

Having developed this rudimentary model, based in the TCA, the open coding and constant comparison of the transcribed small group interactions (as well as other data), I use it in the following chapters to analyse episodes of utterances in an iterative process to identify further analytical themes and patterns of interaction. The charts in Appendix B seek to illustrate some of the grouping of initial open codes that led to the development of the categories discussed in this chapter.

The initial codes, categories and model articulated in this chapter were carefully constructed from analysis of the data collected in this research, including participant observation, research memos, teacher interviews, audiovisual recordings of small group interactions and transcripts. The analytical tools and categories developed were also influenced by my background as a teacher and a researcher, and also by theoretical ideas from Habermas' TCA. This work, which can be considered (broadly) as a set of analytical statements from the perspective of Bassey's (1999) approach to case study, are taken back to the data in an iterative process in an attempt to refine them and generate new insights. This iterative process will lead to the development of findings that address the research questions in Chapter 10.

Chapter 6, next, shows an example of the use this initial work to analyse an episode of utterances, and shares some of the analytical themes that were identified for further investigation in the iterative processes of analysis.

Chapter 6: Analysing patterns of intersubjectivity in small group interactions

Having developed a theoretical framework for analysing intersubjectivity in Chapter five, the next step is to use this framework to analyze the content of episodes of utterances. This interpretation generates analytical statements in the context of the intersubjective framework are used in the iterative process of case study analysis in Chapters 7, 8, and 9. This chapter presents a short discussion of mathematical thinking as communication and Habermas' (1985a, 2001) ideas about the inherent rationality of communication. This is followed by an example of analysis of an extended episode of utterances in a small group interaction. This example seeks to make more transparent the way in which codes, categories and models developed in Chapter 5 are used to identify critical themes for further analysis.

Intersubjectivity in Habermas' TCA is entailed in the structure and function of utterances in interactions. Habermas' (1985a) theory seeks to locate a certain kind of rationality as a prerequisite for communication, and these prerequisite features are entailed in his ideas about formal pragmatics. This set of ideas is epitomised by his counter-factual concept of an ideal speech situation. This concept entails prerequisites for achieving communication including: the notion of the telos of language as fundamentally aimed at achieving mutual understanding; the tripartite nature of the dimensions of validity to which language must refer; the implicit obligation of participants to provide justification for their utterances if challenged; and the conditions under which these challenges can be resolved such that consensus is re-established.

The model developed in Section 5.4.2 incorporates several of these themes while leaving others implicit, such as the relation between understanding and meaning. Another facet left implicit is the double structure of language (cognitive and communicative). In this idea each utterance is conceived as having the structure Mp, in which 'M' represents the communicative dimension (the illocutionary force) and 'p' represents the propositional content which is characterised as the cognitive dimension (Habermas 2002, p xiii). The meaning of such statements is interpretively achieved in interaction with interlocutors that have the capacity to reach mutual understanding. The distinctions developed in the codes and categories can be seen as attempts to make clear, in my interpretation, what is being emphasised in particular utterances in the context of the episodes. In this way, acting as a 'virtual participant' (see Section 3.1.4), I seek to interpret the meaning-making at play in the episode of utterances analysed in this chapter. This work is an attempt to create some transparency of my interpretive analytical work at this stage in the iterative analysis of the case study data.

Moving forward with the analysis of the data, the model has been augmented and refined in analytical reflections. The final aim of this process was a model addressing the situated aspects of communication as well as the technical aspects of linguistic intersubjectivity. The end result, which is presented in a unified manner in the findings of this study (Section 10.1), is a model that serves as a framework for dealing with the analytical themes that are emergent in all levels of analysis (see Chapters 7, 8, and 9).

This chapter contributes to the overall aims of the study in several different ways. Methodologically the analysis in this chapter serves as an example of the iterated analytical moves described by Bassey (1999), and adopted in this study (Section 3.2.2). It also serves as an example of the use of the perspective of the 'virtual participant' in the analysis of the meaning of interactive utterances as described by Habermas (1985a) and discussed in this thesis (Section 3.1.4). Theoretically, this chapter contributes to the claim that models, concepts and categories developed from the point of view of communicative action can be used productively to understand small-group interactions. Practically, analysis in this chapter serves as a basis for potential design decisions and approaches to practice in mathematics education.

6.1 Thoughts on the use of an intersubjective model of student interactions for analysis

This chapter presents a detailed example of how the intersubjective model of student interactions developed in Section 5.4.2 is used to analyse episodes of utterances and identify further analytical themes. This process is instrumental in developing analytical statements that then feed back into the examination of the data in later chapters. The claims developed in this way are based in analysis of data from classroom observations as well as interviews with teachers and other observational data. In the analysis of small group interactions between students working on mathematical tasks, the aim is to characterise the intersubjective processes of thinking collaboratively and making meaning in a community.

The relationship between intersubjectivity and communication is a key theoretical point for this analysis. From the standpoint developed in Habermas' theory of communicative action, the analysis of utterances as meaningful contributions to communicative understanding presupposes the existence of an intersubjective relationship between the participants. The analytical productivity of making meaning through interpreting episodes of utterances as examples of communication could be interpreted as an indication of the existence of intersubjectivity between participants. The potential circularity of this relationship, that evidence of intersubjectivity rests on analysis of communicative action, which assumes intersubjectivity as a prerequisite for the existence of communication, is a problem. This complication stems from Habermas' views regarding the logic of mind-body dualism, his attempt to build a critical theory that moves beyond such dualism demands a theory of communication which rests on a new 'foundation'. The theoretical category of intersubjectivity is Habermas' attempt to address the cognitive basis of communication. This struggle builds on the work of Adorno (1998, p258) who writes,

> Those formative constituents [notions of subject and object] are not absolute but rather a historical development like the cognitive function itself. It is not beyond the pale of possibility that they could disappear. To predicate their absoluteness would posit the cognitive function, the subject, as absolute; to relativise them would dogmatically revoke their cognitive function.

This quote indicates the critical problem that Habermas may be seeking to address with the use of intersubjectivity as a conceptual foundation for a theory of communication. Thus, from a perspective of communicative action, I suggest that interpretation of an episode of utterances as meaningful presupposes in certain ways the existence of intersubjectivity. Thus intersubjectivity is an essential interpretive background concept from a standpoint of communicative action.

One hypothesis developed from initial analysis is that the interactions between students in small groups can be interpreted as communicative action. Habermas' theory provides a model from which to begin understanding how meaning-making and communication are achieved and how these can be understood in relation to intersubjectivity. The model developed in the Chapter 5 offers the potential to analyse the interactions in this research using the ideas of communicative action.

By elaborating upon this model I seek to address ways in which intersubjectivity exists between participants in the research setting, specifically several mixed-ability year seven mathematics classes using complex instruction style approaches, or aspects thereof. In this manner the analysis seeks to understand how and why this style of instruction, which has been identified as an approach with potential for achieving greater equity and achievement from students (Boaler & Staples 2008; Boaler 2006; Boaler 2008), operates from a point of view of communicative action.

In order to make this argument I analyze transcripts of small groupwork and show how students interact around problem-solving to coordinate actions and achieve meaningful insights into mathematical tasks. What follows is an example of analysis of one of the transcripts that I coded repeatedly as the coding schema developed through the process of open coding and constant comparison. Crucially, in these interactions in the data, students engage in patterns of raising ideas, making statements in relation to the tasks and having these ideas and statements challenged by their peers and teachers. These challenges are responded to and the development of insight and meaningful understanding of the tasks is interpretable in the back and forth between the participants. This process was also seen to break down at times, while at other times there was little disagreement between the students and most of the statements were uncontroversial within the context of the problem-solving interactions.

By analysing the way in which this meaning was developed from an intersubjective point of view, fundamental questions about the nature of the practices in which the students were participating are addressed. This allowed insights into the situated nature of the learning and teaching that was taking place. The analysis in this chapter points to the ways in which small group practices on the small group level are connected to the technical linguistic moves that are evidence of a communicative intersubjectivity. This connection, between practices, linguistic moves, and intersubjectivity, leads to practical ideas about pedagogy that may inform the body of professional knowledge (see discussion of findings in Chapter 10).

There is a complex relationship between the normative elements of the interactions, the elements that are predominantly focused on the mathematical and empirical content of the tasks at hand, and the authenticity of participation. Having a model for how these kinds of interactions can entail meaningful intersubjective cognition allows recognition of such intersubjective cognition in data analysis. It also allows for the analysis of what is at play when this intersubjective cognition is not realised or not fully realised or even erroneously realised.

6.2 An example of detailed analysis of one transcript of groupwork

In this section an example of detailed analysis from early stages of the iterative approach to case study analysis is presented in an attempt to illustrate how analytical statements and themes were generated for further exploration. This analysis is adapted from an extended analytical memo focused on analysis of one episode of utterances in the transcript data of a small group interaction from Ms. Phelp's class at Griffin Court College (see Section 4.3.2). All transcript excerpts are identified by a code that refers to the video data from which it is drawn and indicates teacher, school, date and task (see Section 3.3.3). As previously noted, all names used in data excerpts are pseudonyms.

The methodological orientation of the researcher in this analysis is one of virtual participation, which is an interpretive stance in which the researcher adopts the attitude of a participant in the communication being interpreted. In this sense the researcher can take a position on the validity claims entailed in the utterances that constitute the interaction and seek recourse to the researchers own background knowledge in order to make sense of the meaning of the utterances in communicative interactions. The analysis of this transcript also demonstrates evidence serving as the warrant for the legitimacy of this claim (to virtual participation) in that the researcher actually participates in some of the interactions, and is thus more secure in making the claim of belonging (or being able to belong) to the community of practices encountered in the field (see discussion in Chapter 3 Section 3.1.4).

In the task analyzed in this chapter, the students are asked to investigate the set of numbers from 1 to 100, examine how many factors each had and come up with conjectures about why different kinds have certain numbers of factors (described in Section 4.3.2 and appendix G). In the following transcript we see the front-loading of coordinating utterances, which lead to problem-solving utterances. The exchange in Excerpt 6.1 is characterised by coordinating utterances. The first set of bracketed codes indicates the original open coding and the second set of bracketed codes is the coding using the categorical schema developed in the Chapter 5. Note that the excerpts in this section are presented in rough chronological order and taken from a single episode that is particularly

Harry goes on reading the task card until 3:11

Harry: ok does everyone understand? [Question: others' understanding] [QD; QC]

Thomas: we understand- now we want to move on [Statement of understanding] [SD/SC]

Harry: ok you understand [pointing at Thomas]; do you understand [pointing at Charlotte], Charlotte– off in a different world [Checking understanding; accusation of non-participation] [QD/SC]

Charlotte: no I'm not [Response to accusation: Objection] [RC]

Harry: ok Daniel do you understand? [Question: others' understanding] [QD; QC]

Daniel : yes I do [enthusiastically] [Response to question: affirmative] [RD; RC]

Thomas: I reckon we should divide into two so some people work on this and some people work on the other question [Statement: suggestion for division of labor] [SC]

Harry: This is a bit embarrassing Dan - ok I need a pen – ok you're sure – ok investigate the number of factors different kinds of numbers have – ok so Rafael [Orientation Statement] [SC]

Figure 2 Excerpt 6.1 Transcript 22062009GCMPFACTORSFP6

useful in the density and variety of analytical statements generated.

The students were first making sure that they understood what the task was and where they might begin to engage with it conceptually as a group. This engagement in the negotiating of common goals is a key feature of the model developed in Chapter 5. An interesting note is that in line 7, Harry challenges the appropriateness of Charlotte's participation, to which

Charlotte responds defensively. This illustrates the overlapping functions of communication, entailing the coordinating function of speech side by side with the normative aspects of speech. There were power issues at play as in the Active Action [AA] in the next excerpt (Figure 3 Excerpt 6.2 line 28) where Thomas attempts to take control of the process and asserts status implicitly by denigrating task difficulty as well as a potentially correct contribution from another student. This raises issues about power in the context of a Habermasian framework. Questions such as 'What role does power plays in communication?' influenced the development of the next analytical statement:

Status identities can interfere with intersubjective communication.

There was more coordinating after this exchange and then the students began to engage in a series of problem-solving statements (Figure 4 Excerpt 6.3) identifying factors of numbers. Characterizing some of these statements as problem-solving is not quite accurate, as they are more properly constative statements (e.g. lines 46 to 58) that form an essential part in the heuristics of problem-solving.

Thomas: You and Daniel are going to fill out this [holding up the factor chart] and try and get as far as you can; it's very simple does everyone understand what a factor is? [Taking control of process: Division of labor; Statement: denigrating task difficulty; Question: others' understanding] [AA; QD; QC]

Charlotte: numbers that go into [Response to understanding check] [RD; RC]

Thomas: no it's numbers that multiply together to ; so one… let's just do the first ten [Statement: denigrating response; taking control of process] [SVCh; AA]

Harry: ok [SC]

Figure 3 Excerpt 6.2 Transcript 22062009GCMPFACTORSFP6

These insights into the role of constative utterances were integrated into the model of intersubjective communication. This was an area where refinement of the schema grew out of analysis in an iterative fashion and was noted as analytical statements were developed for going back to the data to address this issue, such as:

Many utterances coded as problem-solving are better characterised as 'constative'.

These statements were more task-oriented than they were problem-solving. To what extent was engaging in a technical task (e.g. identifying factors) a problem-solving utterance? From a communicative stance, the constative aspect of these utterances would be their communicative function, while the claims about mathematical objects would be their propositional content. From this point of view, the problem-solving description involves a complex interconnection of practices of argumentation. This is important because the intersection of formal argumentation of mathematics and the everyday argumentation practices of agents communicating within their local lifeworlds is a key educational domain. An important goal for analysis is to recognise this intersection and describe how the rationality of everyday communication can be built upon to develop the practices necessary for participation in formal mathematical discourse-communities.

One of the potential flaws in the design of the task or its implementation may have been to provide the students with a chart for recording the factors of different numbers, indicating that that the task was primarily to identify numeric factors and record rather than to examine deeper questions of how many factors different kinds of numbers have. This came up as a potential problem in the post lesson interview with the teacher and also in other group interactions. This may be related to design decisions that sought to focus student

Harry: ok the factors of number 1 are 1 and that's just one [Activity: identifying factors] [SPS]

Charlotte: ok 2 is 2 and 1 [Activity: identifying factors] [SPS]

[some discussion about the camera + embarrassment]

Charlotte: number 2 [Activity: identifying factors] [SPS]

Harry: 2 is 1 and 2; 1,2,3…1,2,3,4 [Activity: identifying factors] [SPS]

Daniel : aw this is gonna go on for ever [Objection to approach; expression of frustration] [SD]

Harry: 1,2,3,4,5 ? [Activity: identifying factors; mocking tone] [SPS; RD]

Charlotte: no – you have to think seriously about it….[Statement: Demand for legitimate participation] [RD]

Harry: what? -these are all the numbers that can go into …. [Response: defense of legitimacy of participation] [RD]

Figure 4 Excerpt 6.3 Transcript 22062009GCMPFACTORSFP6

problem-solving in the direction of certain trajectories of learning. In the next excerpt (Figure 4 Excerpt 6.3) normative issues come to the fore in line 61.

In Excerpt 6.3, Charlotte challenged the legitimacy of the participation of another student. This example of how power relations within groups of students can be expressed suggested a host of complex things in a very short space: that the Charlotte was able to tell that the other students were not participating legitimately; and that based on this recognition Charlotte felt the authority to challenge the participation of the other students and demand that they 'think seriously' about the task. This was originally coded as a SC, a [statement: coordinating], but it was also a demand for legitimate participation and thus was at some level about validity-discourse in that the student making the coordinating statement was tacitly challenging the authenticity, or subjective honesty (of intent-to-learn), of the contributions of the other students. This dual nature of the speech act, namely coordinating and discursive with regards to authenticity, was developed into an analytical statement:

Utterances can act on several levels simultaneously in intersubjective communication.

This also raised another important aspect of what this analytical model is suggesting, namely that the consensus that is the basis for intersubjective meaning is often tacit and is seen through the absence of challenge to utterances by the participants. What is the impact on the development of intersubjective understanding when there are issues of power and status at play amongst the participants? If participants do not feel comfortable to challenge an utterance due to the perceived authority of the speaker as opposed to the perceived authority of the utterance itself, how does this affect the nature of intersubjectivity and the meaning that is developed? The issue of tacit challenges, tacit agreement etc., is not one that can be ignored, but rather one that must be addressed critically. How can the tacit aspects of intersubjective communication be addressed? An analytical statement to address this issue in the data was formulated as:

Tacit aspects of the preconditions for intersubjective communication are essential for achieving understanding.

Conversely, when understanding is not achieved the reason may also be these tacit features of the pragmatics of communication. In the analysis of the data in Transcript 6.4, the coordinating utterances continue to dominate the episode before transitioning into utterances more focused on problem-solving. What is meant by problem-solving in this case? A different term or a clarification of the definition for this study seems necessary. There are many technical utterances, for instance asserting as a mathematical fact that a particular number was in fact a factor of another number. Identifying factors as a technical utterance is a part of speech considered by Habermas as a constative speech act in that it is reporting, asserting or claiming. This comes about after the students have spent time negotiating and coordinating their understanding of what it is they are supposed to be doing (identifying factors and looking for principles and patterns) and what that entails (finding the numbers that multiply together or divide evenly into other numbers and considering questions of why different kinds of numbers have different numbers of factors).

At this point in the lesson the teacher, Ms. Phelps, got the attention of the class and directed them to send one person from each group to meet briefly with her. To do this Ms. Phelps uses the roles assigned to each student, asking the 'inclusion manager' from each group to come up. Upon returning from this meeting in Excerpt 6.4, Thomas relates directions to the group that focus them on particular questions and directions in the task materials. This leads to another round of coordinating interactions as they interact around the task materials and reorient themselves. The students are then oriented towards the task, and they have identified the questions in the task specifically in their interactions. The question (what kind of numbers have a certain number of factors) they had identified was more complicated than the actions they had begun to perform in a coordinated way

Thomas: just these two questions – we need to think about [orientating statement] [SC]

Harry: so just these two questions- but which problem? [grabs task card looks at it] the problem – [Question: Clarification] [QPS]

Thomas: that one [pointing] [Response to clarifying question] [RPS]

Harry: oh these [gesturing] [Acknowledgement of response] [RPS]

Thomas: what kind of numbers have exactly three or four factors…. [identification of question within task] [SPS]

Harry: 3, 4, 5 [Reading over shoulder, pointing out five is included as well] [SPS; SD]

Thomas: 3, 4, or 5 factors [RPS]

Harry: and so on [SPS]

Thomas: and so on [holding head] [Confusion]

Harry: so [Confusion]

Thomas: it's very simple to work out 4 though I can't remember why – it's like…oh 1 prime numbers only have…3 factors [looks uncertain] and that is wrong it's probably one or something no it's three [seventy three?] do you know any prime numbers with…. [Attempt to relate task to prior knowledge] [SPS; QPS]

Harry: what? [Statement of Confusion] [QD]

Thomas is looking at chart trying to figure something out [orientation towards task resource] [AC]

Figure 5 Excerpt 6.4 Transcript 22062009GCMPFACTORSFP6

(identifying factors of each number) before the teacher intervention. Perhaps unsurprisingly, the students are met with some confusion (Excerpt 6.4).

This change of goal raised new challenges for the group. Thomas attempts to relate the question to previous knowledge (line 96), but seems a bit confused and this is not lost on the other students as Harry expresses confusion towards the utterances of Thomas (line 101). This statement of confusion is analytically noteworthy, because originally it was unclear how to categorise statements of confusion. However in line with the thinking above regarding tacit challenges, statements of confusion are interpreted as entailing a tacit challenge to the intersubjective legitimacy of the utterances of another participant. The potential for intersubjective communication in the interaction is illustrated by the analysis that a statement of confusion can act as a prompt to the original speaker to clarify their contribution or try to understand what it is that their peer is confused about. Such an interpretation would suggest that there is an expectation that things ought to make sense within the group. If a member of the group loses this sense of sense, then work is done to try and re-establish it. An analytical statement was developed around this idea:

Confusion can act as a validity challenge in intersubjective communication.

Conceptual utterances that fit into the schemes of meaning to which the participants have access are one facet of the kind of intersubjective communication in these interactions. The process of challenging each other and responding to each other's challenges is another aspect. These are validity-discourse practices that are essential for the development of intersubjective meaning. Why is it important to achieve the development of intersubjective meaning at all? If one student has more developed sense of factors and mathematics in general why should s/he bother to engage in this intersubjective dance? This pertains to the issue of the primacy of culture versus the primacy of the individuated subject in the development of conceptual understanding. If the subjective agent is seen as the primary source of meaning then there may be little reason to engage in these kinds of interactions. However, if the social is seen as primary in the development of conceptual understanding then being able to engage in these kinds of communicative practices is of vital importance. Alternatively, if the agent and the social are seen as interdependent in the development of understanding, then these kinds of practices could still be interpreted as being vital.

Alternatively, agency and social forces may be interdependent. From a socio-cultural point of view, a student without facility in the practice of achieving tacit and explicit fit of conceptual understanding with others in a social setting through language use would be severely limited in her/his ability to gain more from the cultural font of knowledge. This is far from suggesting that the subjective agent plays no role in the development of conceptual meaning. But equally it is clear that individuals do not (solely) continue to reinvent the wheel. The process of education is at least partly about gaining access to the knowledge and practices of society and that central to this is the ability to achieve tacit and explicit conceptual agreement in linguistic interactions. Intersubjective communication plays a central role in how agents gain access to cultural knowledge. An analytical statement was developed to address this in the analysis of further data:

The practice of intersubjective communication plays an important role in accessing cultural knowledge in a meaningful way.

Following Thomas' attempt to relate the task to prior knowledge and Harry's statement of confusion the group moved on with the task. There was a discussion of prime numbers, and tentative conjectures that prime numbers have only two factors. In Figure 6 Excerpt 6.5 an exchange about prime numbers is interrupted by the intervention of the researcher (acting in a participant-observer role as a teacher's assistant), challenging the validity of something that the students had written down earlier.

Beginning at line 109 the interlocutor (GK) attempts to maintain a discursive stance in the face of Harry's reaction as he starts to change his position (line 124) based on the authority of the interlocutor (as a teacher/researcher). Asking Harry a probing question leads to an articulation of the reason behind his change in position that came from his own understanding of the concept of 'factor'. This is an instance of TD (teacher intervention: discursive). It represents an attempt by the researcher to do at least two things at once: to assess the understanding of Harry's perception of a written error; and also to model the process of interrogating the validity of an utterance that has come into question. Harry shows that he is able to engage in this exchange and demonstrates an understanding of his written error based on his own conceptual understanding. This raises again the issue of the impact of power in communication from a slightly different angle: the power inherent in the teacher's position of an arbiter of knowledge. This also deserved an analytical statement:

Teacher intervention can contribute discursively to intersubjective communication.

Teacher intervention risks breaking down intersubjective communication by entering into an interaction as an authority.

While Harry is interacting with the researcher (lines 131 to 145), Thomas continues with his engagement with the task stating more firmly that prime numbers have only two factors. This leads to a productive, if somewhat confusing, exchange focused on the idea of prime numbers having only two factors. Daniel, Harry and GK have an exchange where Daniel makes a statement that Harry quickly jumps to challenge, and then GK makes a clarification on the behalf of the idea that Daniel has raised. Charlotte makes a statement that is incorrect, but which seems to be based on the incorrect work that the students recorded at the beginning of the exercise – before they started 'thinking seriously', and is challenged by Thomas and Harry. Daniel builds on Thomas' statement that the two factors that a prime has are 'one and themselves'.

In Excerpt 6.6 line 174, Harry challenges Daniel's statement appealing to the fact that they hadn't checked all numbers and that there might be cases that do not conform to the property proposed. Thomas responds to this validity challenge, defending the idea that all primes conform to the generalization of the mathematical property. While no justification for this is articulated, Harry seems to drop his challenge as the group moves on. What is at

Daniel : Harry you've got it all wrong [referring to Harry] [neutral tone: critique of work done on paper by Harry] [SPS; SD]

Harry: I've just figured that…[Acknowledgement of error] [RD]

Harry: 4 as well- 4 doesn't go into 5 [Oriented to erroneous work on paper] [Statement: mathematical property] [SPS]

Thomas: it's just one and five in five [Statement: mathematical property] [SPS]

Charlotte: one three and five [Response: attempt to add to mathematical property] [RPS]

Thomas: no three doesn't go into five [Validity Challenge: identification of supporting reason] [SPS/SD]

Harry: yeah it's true [Statement: supporting validity challenge] [SPS/SD]

Thomas: ok then you're on to seven just has one and seven, all the prime numbers have only two factors [Application of mathematical property; statement: mathematical property] [SPS]

Daniel : one and themselves [Response to Validity challenge: defending generalization of mathematical property] [SPS]

Harry: no hang on- that's not true though if you go up and up and up… [Response to Validity challenge: defending generalization of mathematical property] [SVCh]

Thomas: no it doesn't go up all prime numbers just have two factors, [Response to Validity challenge: defending generalization of mathematical property] [RD]

Figure 6 Excerpt 6.6 Transcript 22062009GCMPFACTORSFP6

play here? My interpretation of this exchange is that academic status and power relations are at play with Thomas being perceived as having more authority with regards to knowledge of the discipline and its practices.

This would seem to indicate at some level a breakdown of rational discourse in that Harry raises a legitimate challenge (though incorrect) to a proposed generalization of the mathematical property that all primes have just two factors, and Thomas responds to the challenge without recourse to articulating a rational explanation for his position. How can this move be understood in light of the theoretical framework of communicative action? This could indicate the need for normative coaching on how to respond to contributions from group members, and that in the absence of this there is a tendency for perceived status to become an implicit source of authority for truth. Is this problematic given the fact that in this case Thomas is correct? This is developed into an analytical statement:

Intersubjective communication can breakdown due to the recourse to authority of a participant in communication.

While the recourse to authority could have distorted communicative understanding, in this case that does not happen. In Excerpt 6.6, Harry challenges Daniel's contribution, which is correct, acting as though Daniel has lower academic status (by being quicker to challenge his validity claims) he backs off in the face of Thomas's support of Daniel. My interpretation of this is that Thomas has higher academic status, but his positions on validity disputes are in line with the rationality and objectivity of the discipline of mathematical knowledge, and this seems to preserve rational basis for the development of communicative understanding.

While there is an opportunity for students to engage in the rational exchange of ideas and justifications there is also the potential for these interactions to reinforce models of understanding that depend on external sources of authority for legitimacy. There are examples of this kind of interaction, where rational justification of positions are put to one side in favour of recourse to authority: 'It is true because I know it is' or 'It is true because the teacher has said so', throughout the transcript and video data. To investigate this in more depth a pattern analysis code was created and the transcript data was re-examined through this lens. As an analytical statement this was expressed as:

An emphasis on justification is necessary for intersubjective communication to take place without being distorted by the recourse to external authority.

However it was also interesting to note that Harry challenged Daniel almost automatically. Perhaps the situation was such that Harry perceives Daniel to have low academic status and Thomas to have high academic status. In this case my interpretation is that Harry feels comfortable to raise a challenge when not confronting Thomas's perceived higher status, but then backs down in the face of this status. What does this say about how knowledge existed between these students? This is especially pertinent since it appears that Thomas doesn't find it necessary to justify his defence of Daniel's contribution. My analysis suggests that these may be practices associated with status, power and knowledge, and in some ways they may limit the students' access to knowledge. As long as students are not comfortable to challenge ideas and ask why and expect a justification that makes sense to them, they are at the mercy of those with superior status, who are assumed to have more access to the knowledge of the discipline, for their access to truth. Further, they are hindered in the development of not only their own conceptual schema but potentially also from developing the skills with which they could pursue conceptual development independently.

Thomas: look just do all prime numbers up to a hundred [Action: Taking control of process, directing] [AA; SC]

Charlotte: what are they? [Action: Taking control of process, directing] [QPS]

Thomas: just do all the prime numbers up to a hundred, and tell me if you get stuck. See eleven…. [Action: Taking control of process, directing] [AA; SC]

Harry: up to a hundred?! [incredulous] [Statement: challenging direction] [SVCh]

Thomas: a hundred's not a prime number [Response: attempt to clarify and justify] [SPS; RD]

Harry: oh…all prime numbers…[mollified] [Response: acknowledgement of clarification + drops challenge] [RD; RPS]

Thomas: it's all numbers which can only be divided by themselves which is why that works [Identification of reason for mathematical property] [SPS/RPS]

Thomas: so 7, eleven, thirteen…. [Identification of some prime numbers] [SPS]

Figure 7 Excerpt 6.7 Transcript 22062009GCMPFACTORSFP6

The practice of rational discourse can be jeopardised by the tacit status identities and relationships, which leads to another analytical statement:

The tacit or explicit use of power distorts intersubjective communication.

Immediately following this exchange, in Excerpt 6.7, Thomas moves laterally from being the arbiter of truth to being the one who is in charge of organizing the groups' work. He engages in the active action of attempting to take control of the process and directing the other students to 'do all the prime numbers up to a hundred'. Harry challenges this direction with incredulity at the magnitude of the task, but is mollified as Thomas clarifies that he means only the prime numbers. In response to the direction from Thomas, Charlotte says, 'what are they?' This indicates that Charlotte feels that Thomas either knows which are prime numbers or knows how to identify the prime numbers somehow. This [problem-solving question] illustrates the potential for this kind of interaction in spite of the status issues and the force of bad practice on the identities and activity of students. When considered with the response that it elicited from Thomas, the recourse to reason rather than authority again came to forefront of the interactions. After mollifying Harry, Thomas responded to Charlotte by identifying the mathematical property that established a number as prime; 'it's all numbers which can only be divided by themselves which is why that works…'

This recourse to the definition of the property prompted by the engagement in the problem-solving interaction by the members of the group illustrates the potential for this kind of educational practice at a very basic level. When students are taught to coordinate their actions to tackle non-routine tasks they have to negotiate meaning at multiple levels. This may lead to more opportunities for students to engage in the practices that constitute access to cultural knowledge in an active rather than passive manner. Concern about what advantage there is to be realised from students working in this way might be legitimate, if these interactions are characterised by some students taking positions of authority (mirroring the teacher's role in a traditional setting) and other students falling into passive roles. However, there is evidence here of students actively engaging in making sense of the task. Another related analytical statement is:

Breakdown in intersubjective communication can be overcome through recourse to justification.

For the rest of the episode of utterances discussed in this section I will describe the interpretation as a narrative account rather than continue to present excerpts. Further detailed analysis of excerpts occurs in Chapters 7, 8, and 9. At this point in this interaction the lead teacher approaches the group to determine what they had done and orient them towards the importance of reflecting on their problem-solving and making a plan. This prompts the students to focus on the directions and questions on the task sheet. This leads to utterances and interactions that help the students make progress in addressing the task. Thomas reads a portion of the task sheet out loud and then reflects on it, trying to ascertain to what it is referring. He then determines that the second question is whether you can figure out how many factors one million has without counting them all. In the same breath he ponders out loud, 'I imagine that if got to one hundred and then you multiplied it by a…' This leads to an exchange between him and Harry that doesn't appear productive, but indicates that Harry is trying to follow the ideas that Thomas is putting forth. Then Charlotte chimes in by starting to determine the factors of 100 and the group spends a bit of time figuring out that step related to Thomas's conjecture. Within this interaction there are instances of validity challenges when one student suggested 40 as a factor of 100. This leads to a discursive response [RD] from Daniel, who had been quiet up to this point aside from working with the camera. An analytical statement related to this pattern is:

Teacher Intervention can facilitate intersubjective communication.

Next, in the course of the technical problem-solving, Harry asks an interesting question that illustrates that he has internalised the concept of factor and how to use the concept to test if a number is a factor of another number. The discursive question that Harry raises acts as a demand for reason and justification, and also as a tacit validity claim. Harry's moves in the interaction indicate that he is nervous about his academic status and his mathematical skills but that he wants to be good at it and have high status. In this case this issue leads to a positive outcome from a point of view of communicative action in that Harry engages in discursive practices which prompt more thoughtful utterances from his group and progress in the search for meaning related to the task. An analytical statement related to this was developed:

Status identities can act as motivation to engage in intersubjective communication.

These statements are analytical statements that are generated out of interpretation of the data and then used to focus further, iterative, analysis of the data to clarify concepts and interpretations of the data until they can be meaningfully expressed as findings (in the form of fuzzy generalisations). In many ways the analytical statements here are somewhat indeterminate. This feature allows for productive iterations of analytical engagement with the data. Is it the case that the data can be interpreted as suggesting that 'student identities can act as motivation to engage in intersubjective communication'? Are there examples in the data where this is not the case? If so, how can the different instances be analysed coherently in order to understand why identities may be motivating in one case and not another?

6.2.1 Reflections on the analytical memo example

This exercise in analysis, to comment and write interpretively and reflect on the meaning of the coded transcripts in the context of the coding schema and associated theoretical framework, is productive. It raises questions and associated analytical statements that may be addressed by going back to the data and interrogating it to see if and how these issues can be interpreted throughout the data as per the integrated strategy for analysis outlined in Section 3.2.2 of this thesis.

This example of analysis in the context of the model of intersubjective communication illustrates the potential for using the theoretical framework and coding schema based in

Habermas' TCA to address how intersubjectivity and communication exist in interactions at the small group level. This analysis might be integrated into a situated analysis of the affordances and constraints that the teaching approaches in the participating classes engendered. This would have the potential to suggest a connection between certain approaches to teaching and the associated analysis related to intersubjective communication. That certain kinds of teaching are conducive to intersubjective meaning making and that this facilitates (or is at least related to) individual development in mathematics learning could then be argued. This is not done in this thesis. Less ambitious, yet still significant, is to work towards such an argument by focusing on how intersubjectivity is interpreted in these cases and to develop a framework for understanding technical expression of intersubjectivity and communication in small group mathematics interactions. This thesis focuses primarily on the second goal, though there are some claims made in Chapters 7, 8, and 9 that address potential connections between a technical analysis of validity claims and utterances in small group interactions and the practices which participating teachers employed.

6.3 Examining evidence of intersubjectivity in small group interaction

As the example in Section 6.2 demonstrated, there is evidence which can be interpreted as intersubjective communication in the transcripts of the small group interactions. However, in many ways the communication is distorted or threatened by issues of power and status. In further analysis, analytical statements developed from initial stages (such as the above example) are used to interrogate the data further and connections are formed between the themes of intersubjectivity in the transcripts of small groups and the contextual data of observation, interviews, and other data.

6.3.1 Identifying thematic issues from the data

The analysis described in this chapter serves to highlight a series of thematic issues for further analysis in the data, including but not limited to: power; status; tacit elements of consensus; the role of justification; the relationship between communication and cultural knowledge; aspects of communicative breakdown; and the role of identity in motivating communication. The issues raised are highly interconnected around the theoretical framework developed in the course of the initial stages of this analysis. Their exploration defines the limits and merits of the theoretical framework developed in this thesis. How can the fragile connection between evidence of intersubjectivity and the analysis of episodes of utterances as communicative action be secured through the theoretical perspectives developed in this thesis? How can the analysis of episodes of utterances as communicative be reconciled with the tacit and negative evidence of consensus in small group interactions?

Another set of important issues is raised by this analysis in the interpretation of distorted communication in the data. Related to the issue of distorted communication are the issues of identity with regards particularly to the impact of academic authority on small group communicative interactions. This points towards the complex relation between power, normative expectations, and justification in the constitution of communicative meaning. These facets of analysis are explored further beginning with an analysis of transcript data as evidence of communicative action in Chapter 7, then a discussion of the role of status and power in the distortion of communication in Chapter 8, and finally an exploration of the potential for a more ideal speech situation in mathematics teaching and learning in Chapter 9.

6.3.2 Development of analytical statements based on thematic patterns

Analytical statements should be developed initially to try to address the research questions and then they should be tested and refined against the data (Bassey 1999). In this iterative process some of the statements are discarded while others are refined. The ideas about intersubjectivity developed in the last chapter and used in this chapter also act as analytical statements and are tested against the data at a level of pattern analysis. Some analytical statements, as discussed in this chapter, are tested and refined through the integrated approach to analysis outlined in Section 3.2.2, can be found, along with their pattern codes, in appendix C.

The ideas developed in this process of analysis require the background knowledge of the analyst acting in the capacity of a 'virtual participant'. The product of this is the analytical statement, which will be refined and finally formulated as a 'working hypothesis'. Thus the product of analysis is based in the empirical data of the transcript but goes further by using abductive reasoning about potential conditions necessary for the data to be as it is. This process of hypothesis building and refinement is a major focus of this work.

6.4 Final thoughts

The analysis in this chapter is a form of testing the models and previous analytical statements against the data. Their productivity in illuminating new analytical issues serves a validating function. The new analytical statements generated here contribute to a warrant for the potential generalization of the previous statements as well as new points of investigation of the data. These analytical statements serve as concepts based in the findings that focus and direct the gaze of further research and analysis in Chapters 7, 8 and 9.

The productivity of the analysis in this chapter serves a validity function with regards to the earlier stages of analysis. The codes, categories and initial model in Chapter 5 represent analysis carefully crafted from classroom observations, analysis of transcripts and video data, participant observation, reflective memos, teacher interviews, professional background knowledge of the researcher, and theoretical ideas from the TCA. The use of these analytical tools in this Chapter and Chapters 7, 8, and 9 seeks to address the research questions in a manner which makes available for assessment by the reader what the interpretive claims are and how they are arrived at. The use of these tools in iterative analysis also allows for interpretive development of the ideas over the course of the research. This is also a validity process, wherein the ideas generated are tested against the data and refined. This is in line with Bassey (1999), but I would also argue that it is in line with Habermas' (1985a) ideas regarding the assessment of social theory.

In Chapter 7, the analytical statement, generated in the example given in this chapter, that student interaction can be interpreted as communicative action is expanded upon. Chapter 8 examines ideas of distorted communication in episodes of utterances in small group interactions, and Chapter 9 explores the idea of approaching something like an ideal speech situation, what that might look like in the context of the small group interactions and what relationship it might have to the pedagogical approaches the teachers in this research are adopting.

Chapter 7: Analyzing participant interactions in Complex Instruction mathematics classrooms as episodes of communicative action

In Chapter 6, analytical statements were developed to further analyze the transcript data from the study. This iterative process of analysis continues in Chapters 7, 8 and 9. In this chapter, the analytical statement derived in the last chapter, that episodes of student interaction can be interpreted as Communicative Action, is further investigated. This chapter seeks to illustrate how this analytical statement was tested against the data and refined into working hypotheses. There are three main elements of communication that are addressed: coordination; problem-solving/constative; and discourse.

7.1 Examining the evidence: coordination and understanding

This chapter examines excerpts from several episodes of small group interactions. Considering some examples and analyzing them from the perspective of communicative action establishes that the data support the interpretation of interactions in complex instruction situations within the study as communicative action and explores to what degree these interactions can be said to entail understanding (or lack of understanding) from this perspective.

In the next sections two excerpts from transcripts are analyzed using the codes and models developed in the previous chapters (see Sections 5.4.1 and 5.4.2) as well as the concept of communicative action discussed in the literature review (Section 2.1.2) to establish the legitimacy of using a lens of communicative action in the further analysis of the data. The analysis of the data focuses on two main issues, the coordination of action, and evidence of understanding through the achievement of goals.

One important thing to note here again is that the category of problem-solving is one that is made up of primarily constative statements (a statement declaring something to be the case) in the context of working on the problem. This category is almost exclusively the domain of the students as well in the data, as the teachers were focused on helping the students without doing the work for them. Teacher contributions were predominantly in the form of coordinating utterances, and discursive modelling. These codes and categories are the analytic framework which drive the analysis of small group interactions as being potentially understood as instances of communicative action. However in order to understand how these discrete codes can help to shed light on the understanding that students are developing in the course of engaging collaboratively, I seek to show how these different types of utterances work together to allow productive collaboration.

With this in mind, recall the model of student interaction developed in the context of the codes and Habermas' Theory of communicative action (in Section 5.4.2 and Section 10.1). This intersubjective model of student interaction articulates an idea of how the various types of utterances worked together to establish common goals and strategies, and to then undertake the actions and utterances that would constitute the achievement of the goals through the strategies. The validity-discourse element in this model came from the breakdown of the consensus that underlies the interactions that lead to achieving the common goals (although it should be noted that this discursive activity can happen also in the negotiation of common goals).

As noted previously, the term 'discourse' is used in a much narrower sense in Habermas' work. In this thesis, I use the term 'validity-discourse' as consistently as possible to signal when I am referring to Habermas' notion. It is important to note that, following the logic of the ideal speech situation as a necessary precondition for communicative action, all participants in a communicative interaction are considered as essentially the same; teachers are considered as having no special recourse to authority except through their ability to bring to bear the 'unforced force of the better argument'. This is one of the main sticking points of this model as this is not typically how power relations exist in classrooms. It highlights a major problem in the use of the TCA in addressing formal educational situations and will be addressed further in Chapter 9. In the next sections, ideas of coordination and understanding are used to re-examine excerpts of transcript data to demonstrate how they can be used to meaningfully interpret the small group interactions.

7.2 Coordination of action

There are many examples of students coordinating their actions around completing their assigned tasks in the small group interactions in this study. For some of the tasks this is more complicated than in others, but it features in almost all of the data. This is indicative of the participating teachers' use of groupwork in their classrooms and may be related to the manner in which a complex instruction style approach seeks to delegate some authority for management of collaboration to the students. The tasks used in these lessons tend to be less straightforward than discrete mathematical exercises often encountered in school-mathematics.

In the first interaction examined below the students are working on a problem[24] involving comparing graphs of four different Olympic races that show axes marked as distance and time but have little other information. The students are directed by the task to interpret what they think is going on in their graph, and then to compare their graphs and interpretations as a group in order to determine which graph goes with which race and what their reasons are for these decisions. We focus on two aspects; 1) That the students are coordinating their actions to achieve the goal outlined by the task;
2) that they do this by reaching and acting upon a number of understandings of the mathematics in the task.

Emily does not say much but is paying attention often. [QC]

Chloe: So what do we do first? [SC]

Chloe: A team of commentators – each person is given a graph to look at…[SC]

Jack: Once you've looked at your own graph you look at them in a group [SC]

Megan: But we have to look at them on our own first [SC]

Jack: OK [RC]

Jack: I don't really get it [SVCh]

Chloe: we need to think about – who won the race, were they in the lead all the way, are there any points where the graph changes what does this mean? Can you make any comparisons between the different <unclear> [SC; SPS]

Megan: On my one athlete B is in the lead the whole way [SPS]

Jack: So athlete B…. [RPS]

Chloe: I don't know cause they all look the same really, cause they all like finish at the same time. [SVCh; SPS]

Other students: unclear

Figure 8 Excerpt 7.1 Transcript 06072009SSMSOLYMPICGRAPHSFP4

The above episode (Figure 9 Excerpt 7.1) details a stretch of student interaction that is mainly focused on identifying the goals of the task and coordinating their actions around achieving those goals (the interpretation and comparison of the graphs). In the process they discuss and use ideas about the mathematical properties of the graphs to interpret the graphs. What is noteworthy is that the students engage primarily in coordinating statements in this excerpt. The utterances reflect that the interaction is focused on figuring out what to do in the task. In line 15, Chloe is referring to the task card.

[24] Chapter 4 Section 4.3.1 Appendix D

The questions on the task card, read by Chloe, lead Megan and Jack to begin making some preliminary problem-solving statements. Notice how confusion plays a discursive role, both initially in a relatively shallow way in line 13 when Jack says, 'I don't get it'. And then later when Chloe comments on the similarities between the graphs in line 23. From the point of view of the intersubjective model of student interaction, the students are engaged in the first steps of coordinating their action around the goals of the task. These goals are being identified and their meaning negotiated collaboratively. These statements constitute the consensus around their strategy and the coordinated action to achieve the goals of the task (in this case the interpretation of the problem). This contributes to an illustration of how communicative action can entail the achievement of goals through reaching intersubjective understanding.

In the next interaction examined (Figure 10 Excerpt 7.2) the students are working on a task to help a hypothetical silversmith by finding patterns that will help the silversmith to know how much silver he will need to make circular bracelets for any sized square box, given that he wants the bracelets to fit into the box snugly touching the sides. While the problem[25] here was designed with the idea that the students might make conjectures about the relationship between the diameter of the circle and the circumference, it was designed so that the students could address the question in its literal complexity of how much silver would be needed. Clues about things like the guidelines for dimensions of various rings and bracelets and the weight of various amounts of silver were put on clue cards that were handed out to the students when they came up with questions. Again the focus of the analysis here is on the way in which the students coordinate their activity to achieve the task goal, and how in the process of discovering a pattern they engage in mathematical practices that can be said to entail their understanding of the pattern they discover. While it could be argued that the task is somewhat poorly designed (or even 'spuriously realistic'), it had some redeeming qualities and the students engaged in authentic mathematical practices in the course of the lesson including asking questions, measuring geometrical objects and looking for patterns in data.

[25] Chapter 4 Section 4.3.3 Appendix H

Mrs. Boxer: What's your task? [TC; QC]

Callum: To get it. To be able to get them circles, like, the bracelet, into a box which is touching the sides. [RC]

Mrs. Boxer: OK, so I know that the bracelet was meant to fit in the box like this, but what was your task? [TC; QC]

Callum: em….

Matthew: Find any patterns or rules. [RPS; RC]

Mrs. Boxer: So, patterns or rules for what? [TC; QC]

Callum and Matthew talking over each other not quite sure

Mrs. Boxer: So read it. Read it off the board, out loud. [TA]

Matthew: Can you find any patterns or rules to help the silversmith with how much silver he needs for a bracelet or ring to fit in any size box. [AP]

Mrs. Boxer: OK, so if for instance how much silver would he need to make a bracelet this size? [QC; QPS]

Matthew: Like the amount around there [indicating the circumference of a circle on the table] [RPS]

Mrs. Boxer: OK, and how much is that? [QPS; TA]

Joseph: Dunno, we haven't measured it. [RPS]

Mrs. Boxer: What's that? [TA]

Joseph: Dunno, we haven't measured it yet. [RPS]

Mrs. Boxer: Ah! OK. [TC]

Samuel : Do we need to measure it? Can I borrow a ruler miss? [QC]

Joseph: you can't measure a circle with a ruler.[SVCh]

Mrs. Boxer: You guys there's all sorts of resources up front. [TC]

Callum: I'm the resource manager [Jumping up to get resources][SC]

Matthew: I'm trying to think how we could do it…[SPS]

Samuel and Joseph in off task talk negotiating for a drink 9:36

Callum: [returning with string and ruler] You measure this line, and then wrap it round the circle to see what the length is…[SPS; SC]

Matthew: yeah I know… [reaching to grab string][RPS; AA]

Joseph: [Wouldn't you] wrap it around the circle first and then put the thing down {garbled} [RC; RPS]

Samuel : you've got to measure how long the string is first [RC; RPS]

Figure 9 Excerpt 7.2 Transcript 10072009GVMBCIRCLESFP3

The episode above (Figure 10 Excerpt 7.2) happened several minutes into the task after the teacher has introduced it. The students at this table are not particularly self-directed, however they do coordinate their actions around both the teachers interventions and each other's actions and statements. The students in this group make a start at interpreting what

Matthew: No you don't…[SVCh; RPS]

Callum: Well, obviously you do [RD]

Joseph: you measure how far it is round then measure how much [garbled] you use [SPS]

Callum: you need to know how long it is [RD; RPS]

Matthew: No you don't [RD; SVCh]

Callum: yeah you do [RD; SVCh]

Joseph: No you don't [RD; SV]

Callum: yeah- cuz you have to make sure how long this is then you wrap it round
[demonstrating with string and ruler and circle][RD; SPS]

Joseph: No- you wrap it round first and then you measure it [SVCh; SPS]

Callum: How you gonna measure it when it's in a circle? [QPS; SVCh]

Joseph: Yeah then you put your finger where the end is [RPS; RD]

This accepted the students get on with measuring circles for a while

Figure 10 Excerpt 7.3 Transcript 10072009GVMBCIRCLESFP3

the task is with lot of direction from the teacher. It should be noted that the teacher did not (quite) tell the students exactly what to do, but rather asks them questions about the goal of the task and how they might start investigating it and then lets them get on with it. The students have an interesting discussion about how to measure the circumference of a circle with a ruler and a piece of string and then get on with measuring some circumferences.

However this 'getting on with it' is very illustrative as well as it shows how negotiation of strategies and goals can loop into validity-discourse. The process of coordinating through the exchange of utterances must be open to this kind of validity-discourse in order to achieve intersubjective understanding. Lines 61 to 82 (in Figure 11 Excerpt 7.3) demonstrate at once the complexity of these seemingly straightforward interactions.

These students are not particularly engaged with the task and they spend a lot of the time in off task conversation, however in the course of the lesson they do a couple of noteworthy things. First, they quickly intuit a common sense work around to the problem: That all you need to do is take a given bracelet and measure the diameter and then make a square box with that length side. This is of course correct and illustrates the highly contrived nature of this problem well. It may also be part of the reason the students were not terribly engaged by this problem. It could also reflect a design flaw, in that the students were given resources consisting of approximately 7 circles of different diameter and three squares that were sized so that three of the circles fit into them. If this had been done the other way round it might have been more focused on constructing circles than on constructing squares, which is one of the things that this group did during its investigation. However, in the course of measuring circles and constructing squares, the students do notice a pattern, so that later when a teacher stops by to inquire into their progress, the students respond with the insight that there is a ratio of about 3 between circumference and the diameter. What is interesting about this is that through coordination of goals and strategies, assisted by the teacher discursively, the students arrive at insights that represent understanding of mathematical content. This serves to further illustrate how understanding can develop through interactive processes interpreted as communicative action.

7.3 Problem-solving and constative

In the original open coding and development of categories through constant comparison, I developed a category described as 'Problem-Solving' with three subcategories; problem-solving statement [SPS], problem-solving question [QPS], and problem-solving response [RPS]. As discussed briefly in Chapter 6, even very early on in my analysis I was aware that this was a crude category given the breadth of research and literature of the subject of problem-solving in mathematics and mathematics education (e.g. Schoenfeld 1994). Upon much reflection I arrived at two insights with regards to the analysis of data in these categories. First of all what characterised most in distinction from the other codes and categories was that this data primarily thematised utterances that were essentially focused information pertaining to the task. In this sense they were constative, in that they were primarily focused on saying something about the problem at hand, whether it be the facts of the task or the ideas (mathematical and otherwise) being brought to bear on the development of a solution (or understanding).

The second point is that this was clearly only part of problem-solving and that the other aspects of communicative action (coordinating and discursive) played critical roles in problem-solving through communicative action. In particular constative, coordinating and discursive utterances, considered within a framework of pragmatic meaning, serve as the building blocks for intersubjective understanding. In the example below an episode of utterances is primarily focused on the facts of the problem and interpreting those facts. It is interesting to note that while the majority of utterances in this episode are thematically constative, they are entwined with discursive statements that act to drive the interaction onward.

Jack: They all like finish at the same time [SPS;SV]

Megan: well yeah because that's the distance [RPS; RD/SVCh]

Chloe: that's the time [indicating the horizontal axis] [RPS]

Megan: So they all ran the same distance, and whoever's further over that way [SPS]

Chloe: first, second, third [RPS]

Megan: yeah- so they were in the lead the whole way, and that person was last and over took them and then they got [SPS]

Jack: athlete B was really slow, and something happened there (gestures at graph)… [SPS]

Megan: yeah athlete… C was a little slow but not [SPS]

Jack: athlete A won on each one… no [SPS]

Megan: Athlete B won on <unclear> [AD; SPS]

Jack: B B A A [SPS]

Chloe:cause that's the time - it's more over that way [AD; SPS]

Megan: yeah - whoever's the furthest over that way is the slowest [gesturing at graph] [RPS]

Megan:the person closest to the distance line is the winner [RPS]

Jack: yeah that's B, see? [SPS]

Chloe: So that person won [SPS]

Megan: yeah cause on this one athlete A was like second an then they went like third and then they went second and then they went third, and in this one [gesturing at another graph] they were last and then they came front one… Athlete B was quite winning for most of them… [SPS]

Jack: Do you [know] what they are? I think that's hurdles – that's four hundred meter hurdles [QPS; SPS]

Chloe: well what's this one? [QPS]

Jack:is that actually filming? [referring to camera]

Megan: yes

Jack: this is a hundred meters [to Chloe] [SPS]

Chloe: how do you know it's a hundred meters? [QD; QPS]]

Jack: because look it's short [RD; RPS]

Chloe: oh yeah it's short [RPS]

Jack: and that one [SPS]

Chloe: that looks [SPS]

Megan: it doesn't matter how far they are… [SVC; SPS]

Jack: that's hurdles that's 1600 meters and that's hundred meters [SPS]

Megan: we have to see like who was in the lead, if they were in the lead the whole way…[SPS]

Jack: What's the other one? [QC; QPS]

Figure 11 Excerpt 7.4 Transcript 06072009SSMSOLYMPICGRAPHSFP4

The next episode (Excerpt 7.4) is from the same group as Excerpt 7.1 above. The students are working on the same Olympic Graphs problem[26]. In this episode, after the students had coordinated with each other around what the task was asking them to do, and how they should go about achieving those goals, the students proceeded to engage in a discussion

[26] As described in section 4.3.2 and Appendix D

and interpretation of the problem. From lines 28 to 49 the students make statements that involve a combination of interpretation and justification, hence the multiple codes for problem-solving statements and validity claims and challenges. In line 47 and 51 the Discursive Action code shows up as the students build an interpretation that addresses the misinterpretation by Jack in lines 28 and 32 (he thought they all finished at the same time instead of running the same distance).

Thus the challenge by Megan and the interpretive exchange by Megan and Chloe which leads to Jack coming into agreement with them by physically gesturing to the axis and articulating the correct interpretation of the meaning of the graph, to which Megan replies in kind gesturing and providing an explanation. In these lines you can see that while the primary theme of the episode is constative (saying things about how the problem is), the discursive themes are still present if not emphasised. This is essentially a minor foray into the validity-discourse 'loop' in the intersubjective model student interaction (see Section 5.4.2 Figure 1 and Section 10.1 Figure 30) with the primary activity characterised by interactions focused on interpreting the problem towards the aim of achieving the goals of the task (which they had already discerned and coordinated around).

This example demonstrates the vital role that constative utterances play in collaborative problem-solving, interpreted through the lens of communicative action. It also demonstrates the way in which these constative statements and questions are embedded within the context of coordinating and discursive utterances in a thoroughly interdependent manner. The consensus formed through interactions is tacit in the absence of potential challenge, and thus prone to communicative breakdown or distortion. This is true both for the level of coordinating around goals and the level of building solutions and interpretations to tasks collaboratively. This leads to the next section in which I take a closer look at how discursive utterances present themselves in the data when interpreted through the lens of communicative action.

7.4 Validity-discourse

Episodes of utterances are also found within the data of this study, and show how the constellations of constative utterances are negotiated. These constellations make up the fragile and changing consensus, which constitutes understanding in the theory of communicative action. These negotiations are characterised by validity-discourse exchanges wherein statements are challenged on one of the three validity bases (objective truth, normative rightness, and subjective sincerity). However, this reveals another problem with the preliminary model of intersubjective model for student interaction (as described in Section 5.4.2). Consensus can break down at any of the first three stages such that discursive demands for legitimate participation[27] can take place in any of the first three stages of the model, which would lead to entry into the discursive loop (stage 4 and 5), and hopefully back to the stage of the model that the students were at before consensus broke down. In the episodes below, two examples of discursive utterances in the context of stage three (focused on solving the problem) are examined. In Figure 12 above (Transcript 7.4), there are two important instances of students entering into discursive negotiation. The first, discussed in section 7.3, has to do with Jack's misinterpretation of the graph's axes. Having resolved that the students continue with the interpretation of the problem.

The students are trying to establish which graph represents which of four different Olympic track events. The task asks the students to interpret their own graph first before comparing the graphs and determining which graph represented which race. Having successfully negotiated basic agreement on how to interpret the graphs in the previous section, the students set out to determine which graph is which in lines 57 to 92. Jack asks a problem-solving question and then without pause offers up a conjecture in line 65 (Figure 12) that one of the graphs is the hurdles. This leads Chloe to respond with a question about another graph (which it seems she believes might be the hurdles). Jack responds by offering up another conjecture that the graph in question is the hundred-meter sprint. Chloe challenges these claims by raising a demand for justification in line 76. To which Jack replies 'because it's short'. Chloe explicitly accepts this justification and the two begin to proceed to interpret the other graphs. Megan seems to raise a challenge to the (common-sense?) justification provided by Jack in line 78, but this is not picked up by the others, and Megan responds by trying to refocus the discussion on the task's goals in line 90. While this is an example of students engaging in discursive utterances in the course of groupwork that can be characterised as communicative action, my interpretation is that the students are building a consensus around a solution which is flawed. This is evidence of a breakdown in productive communication, in that the consensus developed is flawed and not being critically challenged within the group. Issues such as these will be taken up in the next chapter.

[27] In the normative sense of the TCA, rather than the sense of 'legitimate peripheral participation' of Lave and Wenger

In the next episode (Figure 13 excerpt 7.5) a group of students working on the factors task (as described in Section 4.3.1, and found in Appendix G) deals with a misunderstanding around their calculations. In this section the students are figuring out that the factors are the numbers that divide into the integers without remainder or decimal part; they then link this back to the work that Thomas had done originally and determine that it was incorrect (he had apparently been listing all the numbers up to a given number). In my interpretation (from a standpoint of a 'virtual participant') the confusion in this group stemmed from at least two sources, the task and the whole class introduction were not as clear as they could have been, and the group did not spend enough time discussing the problem and coordinating around what it was asking for and how to achieve the task's goals. Yet in this section they were getting on the right mathematical track and dealing with the errors in their calculations. In the course of this they have an interesting exchange around claims and challenges that shows evidence of two brief forays into the discursive loop from the model of communicative action; an exchange in lines 10 through 15 regarding the factors of the number 5 (which seems primarily about computation and understanding of the concept of factor), and a higher-level discussion of the nature of prime numbers and generalizations in lines 17 through 26.

This last exchange is interesting because Harry is making a challenge that there might be a prime number that has more factors than one and itself if you go up high enough (which displays an incomplete understanding of how prime numbers and composites relate to one another). The response from Thomas is interesting because he offers justification based in the definition of a prime number. There is no interpretable acknowledgement of this beyond an absence of challenge and continued focus on identifying factors as a strategy to address the problem. This is an example of how discourse presents itself in the data. It is messy and incomplete, but the potential for collaborative problem-solving can be seen in the discursive negotiation of problem-oriented ideas. This episode is examined further in the next chapter to illuminate some ideas about how this group is struggling to maintain a productive speech situation in the face of complex status issue.

Thomas: ok maybe 2 but [Conjecture: identification of mathematical property, tentative] [SPS]

Harry: what prime numbers have you got? [Question: orientation towards others' understanding] [QD; QS]

Thomas: 3…5….7 [Response to question] [RPS]

GK[leaning over and interrupting]: is two a factor of three? [Teacher intervention: validity challenge] [TD]

Students: oh- no [Charlotte starts erasing] [Response to Validity challenge] [RD]

GK: wait-wait-wait, just answer the question: is it? [Teacher intervention: maintaining discuourse-restating question] [TD]

Students: no [Response to question] [RPS]

GK: ok How do you know? [Teacher intervention: probing question] [TD]

Harry: because you can't double it to make three and you can't [Response to probing question] [RD]

Thomas: So prime numbers only have two factors… [Identification of mathematical property] [SPS]

Harry:– I don't know a way to uh….

Daniel : and there's nothing you can times it by to make…. [Broadening reason to more general justification] [SPS]

Harry: that's not entirely true… [Statement: Validity challenge] [SVCh]

GK: there's no whole number [to Daniel] [Teacher intervention: clarification; TD]

Figure 12 Excerpt 7.5 Transcript 22062009GCMPFACTORSFP6

These examples of discursive utterances and the part they play in interpreting episodes of utterances in small group problem-solving illustrate their vital place in the establishment of intersubjective understanding, and at the same time the tenuousness of this position. In the absence of robust discursive exchanges, the tacit consensuses formed through unreflective exchanges develop into monsters of incorrect interpretations and flawed solutions. From these brief examples we can see the prevalence of such communicative breakdown and the speed with which it can happen. From an educational perspective this suggests that one must consider the manner in which communicative breakdown occurs and what can be done to create the conditions for communicative understanding. This analysis is dealt with in the next two chapters.

7.5 Conclusions and transition

These episodes are evidence that interactions of small groups in classrooms using complex instruction style approaches to teaching mathematics can be seen as indication of intersubjectivity from a perspective of communicative action. This can be seen in the ways in which the students coordinate their goals and actions, the way in which they work together to build arguments and the ways in which they challenge and respond to each other's statements and actions. Further there is evidence based on the analysis presented in this chapter that productive communication, characterised by features of intersubjective rationality, could be seen as entailing understanding of some mathematical ideas and practices and also creating productive contexts for the development of new insights. This latter point requires a definition of understanding based in pragmatics and communicative action. The question of 'How do I know that you know?' is answered by recourse to the pragmatic point that you act as though you hold an analogous concept that is directing your action. Thus the pragmatic point (that if there is no conceivable difference resulting from two conceptions then they can be said to be equivalent) comes into play in the establishment of understanding between people in interaction and thus communication. This point is important as it focuses the analysis of the researcher (and potentially the teacher) on what is actually done and said in the context of the speech situation. The locus for assessing understanding is the speech situation, as opposed to some interior cognitive realm, which would necessarily be characterised speculatively.

The analysis in this chapter serves as another stage in the iterative process of testing the initial Analytical Statements (codes and model), and the second tier pattern-analysis statements, developed in Chapters 5 and 6, against the data so as to develop a warrant for the generalisability of the findings in line with the integrated analytical strategy being used in this thesis. The next two chapters continue this iterative process, re-examining data through the lenses of analytical statements, codes and models developed in earlier stages of the research. Chapters 8 and 9 deal with two important aspects of interpreting small group interactions from the point of view of communicative action, namely the analysis of the breakdown of communication, and analysis of the potential for a more ideal speech situation through pedagogical design and practice.

Chapter 8: Examining evidence of distorted communication and the breakdown of communicative action

In the previous chapter I laid out the analysis supporting the claim that the small group interactions in this study can be productively interpreted from the perspective of communicative action. This analysis was part of the iterative process of testing initial codes and models and second tier pattern analysis laid out in Chapters 5 and 6. In this chapter, having established that these small group interactions can be seen as communicative action, the analysis turns towards consideration of the potential for communicative action to break down or fail to achieve sufficient coordination and understanding to make progress on the mathematical task at hand. In Chapter 6 several analytical statements were articulated that pointed towards the potential for the exercise of power to distort communication. This could happen due to the teacher intervening and acting as a source of authority, or more problematically students could attempt to assert power over other students through their speech acts. There is a subtle yet important difference between the coordination of action between equals and the unilateral direction of goals and actions by one participant. There is also the problem of absent or poorly functioning social normative expectations around how to work collaboratively, which may create situations that are not respectful or collegial.

In Sections 2.2.4 and 2.2.5, some ideas about why evidence of distorted communication may be significant were explored and expanded upon. Two of these ideas deal with how communicative understanding can be undermined: 1) the threat to the development of non-pathological ego-identities that can be found in the concept of systematic distortion of communication; and 2) the threat to non-pathological social integration that can be seen in the concept of the colonisation of the lifeworld. Can these two realms of development (the social and individual), which are interconnected and vital in the field of mathematics education, be understood as related to the moves being made in small group interactions? I turn in this section to the examination and analysis of breakdowns in communication in the data in this study. The rationale for the three episodes that are examined in particular is that they are rich cases of communication that are distorted in different ways. The term distorted here indicates that from the point of view of the theory of communicative action there is an ideal form of communication, which has as its telos the coordination of action through mutual understanding. The first episode is an examination of power issues. Issues of ability positioning and resistance to school mathematics interfere with one group of students working on a problem to determine the number of factors different integers. In the second episode a group realises only partial understanding of the implications of a mathematical task after not achieving consensus with a teacher's attempt at discursive intervention. In the third episode a student, through the use of speech acts which position him as (supposedly) more competent than his peers, distorts access to the rationality of the discipline of mathematics by undermining other students' contributions, and other students cope by, in turn, submitting to this power play, supporting it in order to gain power, and retreating from interaction and using humour to try to mitigate the situation. The next section analyses 4 excerpts in chronological order, examining issues of micro-politics in one small group interaction.

8.1 A case of an excluded boy: issues of alienation and resistance

One particular episode in the data reflects the issue of alienation based on ability labelling and identification. In Chapter four, Section 4.3.2, this episode is discussed in the context of the work done at Griffin Court College. In that episode a boy, Oscar, is explicitly labelled by as 'not being able to understand it' and this is seen as justification for him not to be included in the group interactions around the task by other students, the teacher, and the boy himself. Analysis of this episode (Excerpts 8.1, 8.2, 8.3, and 8.4) shows how the student that is most vocal in the interaction, Megan, positions Oscar quite forcefully as not able. Oscar participates at first, following directions form Megan, but gets frustrated quickly, perhaps by the tone and content of Megan's comments, and starts acting in ways that can be interpreted as resistance to participating in interactions around the assigned task. Oscar starts fooling around, first in seemingly insincere responses to Megan, and then by changing the conversation to a social one about dating and such. Throughout the course of the rest of the groupwork time, Oscar is on-task only when a teacher is directly observing and interacting with him, and even then it is clear that he has not been following the work and interactions done by the other group members.

How is one to analyse this? Megan is quite frustrated by Oscar's behaviour, but Abigail seems to be somewhat entertained and interacts in a friendly way despite sometimes making jokes about how little he listens or pays attention to the class-work. Megan goes as far as to say, "You know what, I'm so glad that the teachers know he's a pain in the butt…" to the other group members.

This issue can be analysed using the theory of communicative action as it is essentially a set of exclusionary practices that deny one student access to participation in communication to varying degrees and thus undermines the pre-conditions for communicative understanding. In mathematics teaching the pervasiveness of ability labelling and the use of hierarchical levelling in the assessment of students are already recognised as problematic (Knapp et al. 1995; Cooper & Dunne 1998; Linchevski & Kutscher 1998). However, what the perspective of communicative action reveals is that these kinds of communicative moves may interfere with students participation in the practices that might allow them to develop their identities as competent students of mathematics. By creating conditions where the contributions of some are seen as less worthy, one student, Oscar, is excluded and discouraged from participating in the back and forth which entails the potential for communicative understanding.

Systematic distortion of communication is a useful frame for the analysis of the small group data in this study. Developing an analysis of distorted communication in classroom groupwork interactions could point the way toward further research that examines the connection between larger institutional features and the practices of the classroom. This kind of analysis is beyond the scope of this thesis, but the analysis in this chapter of some episodes of utterances as evidence of systematically distorted communication may point toward the analytical potential of the concept of colonisation of the lifeworld.

This example of the 'boy who was excluded' also reveals the way in which the concept of systematically distorted communication is used to interpret transcript data. In this example the boy, Oscar, was acting inappropriately. This was interpreted as an intentional strategy to avoid participating in the interactions that reinforced his identity as not being able. In the episode in Figure 14 Excerpt 8.1, we see the display of identity conflict result in unproductive exchanges. These interactions show two things, first the exclusionary moves around Oscar's perceived ability, and second the way in which Oscar chooses to resist through resorting to inappropriate social positioning.

Megan: Have you filled in the bit in the middle?

[Oscar continues to manipulate the markers]

Megan: the bit in the middle, not the bit on the outside

Oscar: this is [unclear- very focused on manipulation of counters]

Megan: You can write it on a bit of… um – Abigail [getting attention of Abigail]

Megan: He can write it on a piece of paper, but when it comes to use the things, he can't do it!

Megan: Oscar- Oscar- You- did – it- on- a –piece- of- paper [synchopated tone as before- circling Oscar's previous work as she says it; Oscar is focused on counters not making eye-contact]

Megan: [repeating and circling previous work continuously] You- did – it- on- a –piece- of- paper

Oscar: Yes- I- know! [Looking up and making eye contact. Speaking in almost similar syncopation- bit defensive?] There we go [gestures to counters] No I'll show you a proper one [wipes the counters into a pile and starts counting again]

Megan: What was the rectangle that you did on the piece of paper?

[Oscar makes the four by five arrangement of counters that reflects the work that he had done earlier]

Megan: Ok pass them to Abigail and see if she can do one.

[Oscar gathers up the counters and moves them towards Abigail]

Oscar: OK

Megan: See if you can do one Abigail

Oscar: Be a good girl Abigail

Megan: make sure you have twenty

Figure 13 Excerpt 8.1 Transcript 24062009GCMPRECTANGLESFP26

Before the excerpt below took place the teacher, Ms. Phelps, had taken the class through an extensive whole class discussion of what the task entailed. As the group starts to work independently around the task, Megan takes control right away. In the excerpt we see Megan directing Oscar. She then proceeds to criticise his efforts and he becomes defensive. She speaks to him slowly as though he was a small child. At first he doesn't respond and then he responds clearly in a defensive manner, using some rather unsubtle sarcasm. This use of sarcasm seems to be a response to Megan's condescension. At the end of this

[ot playing with the camera- Megan takes camera and sets it down]

Megan: OK, Oscar, can you do another one for me?

Oscar: No. Yeah.

Megan: because you did the four by five- do you want me to do one?

Oscar: No I want to do one- I'm a big [button?]!

Megan: Yeah- whatever you say Oscar [sarcastic- but friendly?]

[some other talk that seems interesting but is too unclear to interpret]

[TA comes over and givesthem more counters and moves Abigail next to oscar; all three are now focused on making arrangements of counters]

Megan: What's that?

Oscar: Four by six

Megan: is that <really> twenty?

Figure 14 Excerpt 8.2 Transcript 24062009GCMPRECTANGLESFP26

episode Oscar takes one parting shot saying, "Be a good girl, Abigail." Which seems to be a kind of mockery of the directing attitude of Megan. Oscar is able to do some of the things that Megan directs him to do, such as construct a rectangle with certain dimensions, but quickly becomes frustrated and distracted and begins responding to Megan's directions with social talk. There is a slight transition into silliness, and then some inappropriate posturing (possibly bullying-type behaviour), which is followed by a teacher assistant intervening and getting the group focused back on discussing the task at hand.

This next excerpt (Figure 16 Excerpt 8.3) is from the same episode a few minutes later. Oscar is messing around and hasn't been paying attention to the ideas that the others have been having. Megan is trying to explain the idea that the group has come up with. The exchange with regards to behaviour is very telling, as is the more or less blatant lack of reference to the ideas developed by others in the group in Oscar's responses to Megan's questions. In line 316 in Excerpt 8.3, Oscar makes a statement that emphasises the claim that his behaviour is normatively correct. This is a suspicious claim, but the other members of his group do not directly challenge him on it. Abigail makes a wry comment, which could be interpreted as a challenge to Oscar's claim. Oscar doesn't respond to the comment and focuses his body-language at Megan.

[Megan is trying to explain the conjecture the group has come up with:]

Megan: We still have to think about this. OK, so four factors has four rectangles…

[While Megan is speaking Oscar starts throwing some of the counters. Abigail, the girl sitting directly to his left is messing about a bit too and interacting with Oscar socially a bit. Hannah, off camera to the left, throws some of the counters back at Oscar. Oscar glances away (presumably at one of the teachers who may be noticing the horse play) and says "what are you doing?" [in a mock indignant manner], and then starts to 'pay attention' to what Megan is saying;

Oscar: …yeah four rect- four squa – four sides… [Looks away from megan to his left, then straightens up and looking further into the distance, says:]

Oscar: I'm not, miss! [pause, turns back to Megan] I'm listening… four rectangles….

Abigail: for once [amused]

Megan: equals…

Oscar: …equals…a square!

Abigail: [A look of confusion comes over her face, followed by a silent 'what!?' and shaking head looking back and forth at the other two group members]

Oscar: …equals a bigger rectangle!

Megan: Four factors equals….

Oscar: …a rectangle.

Megan: [pause] How many?

Oscar: [quietly] four?

Megan: Well done.

Oscar: Told you I listened!

Figure 15 Excerpt 8.3 Transcript 24062009GCMPRECTANGLESFP26

Oscar's body-language now suggests he is paying attention to Megan. And he is, in a way. Despite the fact that his responses to Megan bear no sign of his having noticed or understood the work done by others in the group, Oscar engages with Megan in a sort of initiation response evaluation pattern (IRE) quickly figuring out the answer that Megan was looking for in her very discrete closed questions. He then re-emphasises the normative correctness of his participation in line 339. The claim is no less suspicious at this point than it was the first time Oscar emphasised it.

But Megan is not going to let Oscar off the hook for these claims. In the next section (Figure 17 Excerpt 8.4) she tries to expand her questioning to see if Oscar understands the pattern that the others have identified (that the number of different rectangles possible under the conditions of the task is equal to the number of factors of the integer area).

Megan: Two factors?

Oscar: uh, Eight fifteenths rectangles

Megan: One [severely, also incorrectly]

Oscar: Which has two in it.

Megan: No [severely]

[Abigail pokes oscar's arm with a round plastic counter]

Oscar: See?! She cut me! [Holding up arm which got poked]

Megan: I don't care.

Oscar: Do you know what [unclear] is?

Megan: I don't care

Oscar: You're in love with Billy Smith

Megan: No I'm not

Oscar: Who do you love then?

Megan: No one.

Hannah: She's in…what?

Oscar: She went out with Harry Smith

Abigail: OSCAR LIS-TEN !!

Hannah: Did you. [in tone like oh-really?]

The conversation goes on a bit about relationships of different people not in group until a TA comes over and intervenes around group roles

Figure 16 Excerpt 8.4 Transcript 24062009GCMPRECTANGLESFP26

Oscar does not rise to challenge and it becomes unclear what either Megan or Oscar may be referring to. In lines 351 to 359 in the next section (Figure 8.4) a shift in social positioning takes place. This shift takes the form of Oscar shifting the conversation to one of relationships. This puts Megan on the defensive, especially when Oscar mentions that Megan went on a date with a boy named Billy Smith. Hannah gets interested in what Oscar is saying and the whole conversation (the one about the mathematical task that is) is headed for de-rail until Abigail forcefully steps in to curb Oscar's inappropriate comments. The conversation becomes one about relationships for a few minutes until a teaching assistant comes over and intervenes, but is less confrontational and involves all the members of the group.

The episode continues and the group is able to articulate and record the idea about a pattern that they developed from investigating the integer area rectangles. Most of the productive group interactions occur when a teaching assistant or the teacher, Ms. Phelps is present. At the end of the groupwork time Megan and Oscar are engaged again and she asks him to explain the idea of factors in the context of the task (where the factors were the lengths of the sides of the integer rectangles). Oscar uses his own words referring to the side-lengths as lines and gesturing- the answer is not great to be fair, but Megan hammers him for his mistake, again. This is another example of the ability positioning that is related to Oscar's exclusion from the communicative action that led to the ideas developed by the group.

Oscar is being excluded from the communication, and he is also excluding himself. He doesn't use effective strategies for mitigating the negative power positioning within the academic discourse but rather resorts to school-yard gossiping and posturing (potentially bullying-type behaviour). The girls in the group respond to this reasonably well, but there is much less communication going on about the mathematical task. I was acting as a participant observer in a teaching assistant role in this lesson (alongside Ms. Phelps and the teaching assistant mentioned earlier). I intervened to check for group understanding of the task at one point in the lesson, and found that Oscar was not able to follow the ideas that the others in the group were presenting. When I tried to engage the students in working with Oscar to make sure everyone in the group understood the ideas they were coming up with, Abigail said "but he doesn't really care, so…" and Megan said, "He doesn't listen anyway." Which is perfectly true, and it seems as though this lack of what Skovsmose (1994) might call an intention-to-learn has effectively positioned Oscar outside of the school-mathematics discourse. The interplay of power and identity between students in groupwork is not unusual, though this example is particularly stark, at least in the data from this study. I will examine some of the teacher-group interactions as well as classroom culture issues in the next section in the analysis of supporting communicative action around mathematical tasks.

8.2 A case of failure to achieve consensus around teacher intervention

In this section I examine data that supports the analysis that consensus may not be achieved between students and teachers in small group interactions around teacher interventions. This episode signals the challenges that teachers face in participating in discourse while mitigating their positions of power. Is this an instance of breakdown of communicative action? The underlying purpose of communicative action is to reach understanding with other participants as a means of coordinating goals and actions. This idea also relates to consensus theories of truth such as that developed by Apel (see Section 2.1) in that the understanding that is reached by any given group is open to further critique by any other participant. In this formulation communicative action should support understanding that is always approaching truth.

In Figure 8.5, the students are working on the Olympic Graphs Task at Summit Secondary School. This task is described briefly in Section 4.3.1 and in more detail in Appendix D. The teacher, Ms. Somerfield, has a particular answer in mind, but does not want to force it upon the students because she believes that this would interfere with their learning. Instead she tries to intervene by trying to draw students' attention to different features of the task and asking questions that she feels might give them the opportunity to puzzle out the correct solution to the task. However the features of the task that Ms. Somerfield is trying to persuade the students to think about are ambiguous, and this group, though they make some progress throughout the course of the lesson, and participate in the whole class plenary, do not seem to make progress towards a definitive justified (let alone correct) solution during their small group interactions.

Unwilling to take authority for problem-solving away from the students, Ms. Somerfield is unable to get them to reconsider their interpretation of the graphs in the task. The group here goes on to make a decision which fails to take into account the implications of some

Ms. Somerfield : Alright, how we doin? [QC/TC]

Chloe: we've sort of said that that one is the one – five hundred, no fifteen hundred one, that one's the… [RPS]

Jack:hundred meters [SPS; SVCh]

Chloe: hundred meters; that one's the hurdles, did we say? [SPS; QC]

Jack: yeah, hurdles… [RPS]

Chloe: and that one's the other one; [SPS]

Jack and Megan: the relay [SPS]

Ms. Somerfield : Why? [TD]

Chloe: Because that one is longer than all the other ones [RD]

Ms. Somerfield : Right [TA]

Chloe: and normally if you do a long one you go slow and then like right at the end you go fast [RD]

Ms. Somerfield : Sprint at the end, yeah [TA; SPS]

Chloe: and then that one is like going over and then you stop and like you get slower to like jump [SPS]

Ms. Somerfield : Do you though? [TD]

Chloe: you kind of do though because you run and then you have to… [RD]

Jack: or while you're in the air, it slows you down a bit… [RD]

Megan: when you're in the air you can't run can you? [RD]

Ms. Somerfield : but if you watch the hurdles in the Olympics, [TA; TD]

Jack: yeah they…[RD]

Ms. Somerfield : if you think about how they run- they- it's just in their stride isn't it? So they're running really fast, really massive strides and then they're just tschew! [zooming noise] And they don't really slow down…. [TD; TA]

Megan: but while they're in the air they might be slowed down [RD; RPS]

Ms. Somerfield : Maybe… [TD]

Megan:…but it's not much…[RPS]

Ms. Somerfield : But what's the other race that you're looking at? [TD; QPS]

Jack:It cold be the relay… [RD]

Ms. Somerfield : Why would it be the relay? [TD]

Jack: Because while they're swapping over they will lose a bit of time during that bit and not go as far… [RPS]

Chloe: yes so he might have started [SPS]

Megan: and then stopped [RPS]

Chloe: and then he might have gone and then he might of gone [gesturing to the graph] because that's really straight and that's a straight line there with that one… [RPS]

Figure 17 Excerpt 8.5 Transcript 06072009SSMSOLYMPICGRAPHSFP4

of the ambiguous features of the task, namely that the axes were unlabelled and that it was noted in the task directions that the graphs were not necessarily drawn to the same scale.

Ms. Somerfield tries to draw their attention to different aspects of the problem and challenge the reasoning for their interpretations, but the students are not able to take this feedback on board and end up achieving little consensus in the course of the interaction and only partial understanding of the mathematical task.

From the perspective of the intersubjective model of student interaction what is occurring seems to be a failure of the processes of validity-discourse such that students are not able to arrive at a shared interpretation of the task. They have successfully negotiated the goal of the task and have been working towards it by interpreting their own graph as well as its relationship to the graphs of the other group members. In many ways this is very successful communicative action, however their failure as a group to arrive at a correct answer, despite the fact that they focused on justifying their ideas and interpretations, raises questions about what the meaning of 'success' is in regards to episodes of communicative action. Ms. Somerfield tries to support the group towards the development of a correct solution, but lets the students work it out on their own despite her own reservations about their interpretations and associated justifications. The problem-solving diverges and the validity discourse, that ought to allow the unforced force of the better argument to come to the fore, does not happen in a way which allows the problem-solving to re-converge.

Jack: that means that would have to be hurdles [SPS]

Ms. Somerfield : See what you think, because actually you guys disa- it's up to you to decide [TC]

Chloe: I don't really get how the relay would work though, because not all of them would be running at the whole time. [SPS; SV]

Megan: yeah [RPS]

Jack: No [SVCh]

Ms. Somerfield : Well it's a strange one isn't it? [TD; TC]

Ms. Somerfield : So basically on the relay we're looking at the speed of the team, so whoever's running we're looking at that person, so in each section of the graph so for the first section it's the first person; for the second section it's the second person and so on, ok? [TD; TA]

Ms. Somerfield : So you decide- you decide which one it is… [TC]

Megan: So what do we think this one is? [QPS]

Jack: I still think that's the hurdles [RPS]

Megan: No that's the 1500 [SVCh; RPS]

Chloe: and Daniel's is probably 200 [RPS]

Jack: meter hurdles? [QPS]

Chloe: No no no no no…. [SVCh]

Figure 18 Excerpt 8.6 Transcript 06072009SSMSOLYMPICGRAPHSFP4

In the end the group is able to participate meaningfully in the plenary of the lesson and is at least exposed to the correct solution and the accompanying justification. In this way, the communicative action of the small group prepared them to take part in the whole class discussion. Their struggle also informed Ms. Somerfield as to some of the parts of the task that students might struggle with and why, which informed her management of the plenary to (apparently) good effect, the analysis of this plenary will be touched on in Chapter 9 and was described in Chapter 4, Section 4.3.1. The analysis of this episode is illustrative of the fact that communicative action in small groups is not guaranteed to arrive at a consensus, and if it does it does not guarantee that such consensus is correct.

8.3 A case of a boy who denigrated others' understanding

In the next episode Harry acts as though he is an authority, both in terms of mathematical knowledge and also in terms of managing the group's action. Another student, Thomas, seems insecure and is continually taking sides with Harry. This creates a situation where the group becomes dependent for mathematical knowledge on Harry, even though other students have good ideas and sufficient mathematical background to make independent contributions.

Participants in communicative action distort the access to the rationality of the discipline of mathematics by undermining the contributions of other students. In this episode (8.2 through 8.5) the other students cope by, in turn, going along with this power play, supporting it in order to gain power, and retreating from interaction and using humour to try to mitigate the situation.

Thomas: This is a bit embarrassing Dan- ok I need a pen – ok you're sure – ok investigate the number of factors different kinds of numbers have – ok so Harry [Orientation statement; SC]

Harry: you and Dan are going to fill out this[holding up the factor chart] and try and get as far as you can; it's very simple does everyone understand what a factor is? [Taking control of process: Division of labor; Statement: denigrating task difficulty; Question: others' understanding; AA; SC; QD]

Charlotte: numbers that go into [Response to understanding check; RD]

Harry: no it's numbers that multiply together to ; so one… let's just do the first ten [Statement: denigrating response; taking control of process; Unilateral; negative view of others' understanding; SVCh; AA; SC]

Thomas: ok [RC; RD]

Harry: one [SPS]

Figure 19 Excerpt 8.7 Transcript 22062009GCMPFACTORSFP8

Harry is making a power play here from the start, which is one of many indicators that he identifies as a high status student. What is especially troubling about this excerpt is that, right from the beginning, Harry asks a question and when Charlotte answers it with what appears to be the beginning of a correct answer, Harry denigrates the response and offers a different articulation of the mathematical fact. Charlotte does not object to this and continues to work hard to solve the problem. However, as the episode moves on a pattern emerges that shows that the power dynamic established early on in this interaction is obscuring the rationality of the discipline of mathematics. I suggest that this is because of a situation of distorted communication, brought about by the overbearing attitudes and actions of Harry.

Charlotte: what about 9? [Question: orientation towards filling out factor chart; QPS]

Harry: one three….and …nine [RPS]

Charlotte: no, four and five [Alternative claim; RD; SVCh]

Harry: four multiplied by five is…? [Question: orientation validity challenge; QD]

Charlotte: oh I'm thinking of add [Response to implicit validity challenge: agreement; RD; RPS]

Thomas: four multiplied by five? You don't know that? [Question: Clarfication, loaded; Denigrating; Incredulous; AA; QD]

Harry: no I do, I was asking her [Response to question; Defense of status; implicit denigration of Charlotte's academic status; RD]

Harry: it's twenty [SPS]

Harry: so Daniel you have to help her with this [Action: Taking control of process, delegating; AA; SC]

Figure 20 Excerpt 8.8 Transcript 22062009GCMPFACTORSFP8

This is the second time that Thomas has engaged in this way, by displaying insecurity regarding academic status. His statements have a normative content, and appear to be a sort of pecking order type behaviour. Thomas uses incredulity to assert his own status, and seems to be substituting normative power of social status for the communicative, or epistemic rationality of the discipline. My interpretation is that while there may be the potential for interactions that can be interpreted as rationally motivated communication, there is the potential for this communication to be distorted by appealing to social power to substitute for the identification of justifications for statements.

This explicit, continued, denigrating of Charlotte's ability begins to raise serious concern for me as the researcher. There is here a serious potential gender issue, layered onto the other issues of academic, peer and social status. It is important to note that gender is not addressed theoretically in this research. This issue arising in analysis of the episodes of utterances indicates that the theories in this work may be useful in analysing gender in other work. This issue could be interpreted as an issue of social status as well, illustrating the potential for social issues to overlap.

The next part of the episode (Figure 22 Excerpt 8.9) raises issues of consensus and authority with regards to discipline content knowledge. The issue of expressing confusion comes up. The students who have been marginalised make use of expressions of confusion to re-establish communicative participation notwithstanding the power relationships at play. The discussion of what integers divide into other integers evenly involves claims about mathematical knowledge.

Charlotte: four doesn't go into twenty two does it? [Question: orientation identifying factors; QPS]

Daniel: yeah it does [Response: orientation identifying factors, false; RPS]

Charlotte: no it doesn't [Validity challenge towards response; SVCh]

Thomas: yeah it does [Statement: supporting validity claim in s4's response, false; RD]

Charlotte: four? [Question: confusion; RD; QD]

Charlotte: Harry, four doesn't go into 22 does it? [Appeal for input on validity dispute; QD]

Harry: uh [distracted still working by self]

Thomas: oh wait no- no it doesn't [Statement: withdrawal of validity challenge; SV]

Daniel: it doesn't- four doesn't go into 22 [Statement: reversal of previous validity challenge; SV; SPS]

Charlotte: der- [Confusion; SD]

Thomas: six goes into twenty….. [Statement: orientation identifying factors, false; SPS]

Charlotte: eight doesn't [Statement: orientation identifying factors; SPS]

Charlotte: how about 3? [Question: orientation identifying factors; QPS]

Thomas: eight does! [Validity challenge, false; SVCh]

Charlotte: no it doesn't [Response to validity challenge: objection; RD]

Thomas: oh no it doesn't…. no it doesn't- sorry! [Statement: reversing validity claim]

Daniel: you should know better! [Chastisement; Humor; SD]

Thomas: Daniel- you're not doing anything ok? [Demand for legitimate participation; SVCh]

Daniel: I'm filming [Defense of participation; RD]

Thomas: so what? You can still suggest problems and stuff [Response to defense of participation; RD]

Charlotte: serious Daniel you really should [Support of Demand for Legitimate particpation; SV]

Charlotte: I've got it for twenty two- one two eleven twenty two [Statements: orientation identifying factors]

Figure 21 Excerpt 8.9 Transcript 22062009GCMPFACTORSFP8

At the beginning Daniel makes a mistake in line 68. Charlotte was correct in thinking that four does not divide evenly into twenty-two. Daniel answers the confirmation-seeking question incorrectly. Charlotte expresses her confusion in line 74. This acts as a challenge to the position taken by Daniel (and now Thomas as well). Daniel reverses his position on four into twenty-two. Then they move on to talking about other numbers. No reasons are provided for the reversal, which might explain Charlotte's expression of slight confusion in line 85, but she rebounds quickly and continues to collaborate around identifying factors of integers.

In Excerpt 8.10, the students are engaged in an ongoing negotiation of participation expectations, which entails validity-discourse moves focused on normative features of interaction as well as constative statement about mathematical facts. The boy filming and joking around in this episode, Daniel, was not at all lost with regards to the problem. He was able, right from the beginning of the task to contribute strategies and mathematical facts to the interaction. This raises the question of why he chose to remove himself from so much of the interaction and work that the other students did, pre-occupying himself with the camera at times, interjecting with humour at other times, and often simply sitting still and idly taking it all in. This could be interpreted as a an instance of clearing the ground for discourse through the use of humour, as discussed previously in Habermas' ideas of distorted communication.

Daniel: I'm gonna film you guys talking

Harry: n n no just do everyone Dan, just put the camera down - you need to help her do that because… I'm sure she won't be able to do it on her own… [AA; SC]

Thomas: Daniel are you going to be a director when you're older? [Humor; QD; QC]

Harry to Daniel: will you? [Humor; QD; QC]

Daniel: yeah [Humor; RD]

Harry: ok come one- just leave the camera alone, we've got to…[AA; SC]

Figure 22 Excerpt 8.10 Transcript 22062009GCMPFACTORSFP8

So in the end, this evidence of the potential for communicative breakdown has many features which make it seem like potentially positive and productive interaction.

However, there are clearly interactive issues at play here that cannot be characterised as being beneficial (or of an emancipatory or egalitarian nature) for the group or the members that are negotiating their identities everyday in exchanges like this. There is a threat the basis for rational consensus is undermined. Since Harry does not always respect the contributions of others and because he is continually assuming the authority as an arbiter of mathematical truth, others in the group are put in the position of having to doubt their own contributions, even when they are not incorrect.

While justification is evident in this data, often Harry's validity discourse is not supported by explicit justification, thus others are denied access to the rationality of the argument and must depend on an external authority (in this case Harry) to secure the correctness of assertions. This threatens the preconditions for communicative action, and could lead to interactions that display more features of instrumental action, in that mutual understanding is not necessarily the basis for coordinating action, but rather others actions are directed by an individual who holds the goal (and the understanding of how and why to achieve it) to himself and adopts an attitude such that others are a means to the end of achieving the goal. This does not truly describe Harry, and the other students take action in this episode to assert their agency. However, this analysis suggests that the potential is there for communicative action to breakdown in this way, and with it whatever learning might be associated with achieving mutual understanding and coordinated action based on such in the mathematics classroom.

8.4 Conclusion

In this chapter I have addressed three cases of distorted communication. In each of these cases communication can be interpreted as occurring, but in some way the rationality of that communication is called into question. A distortion of communication is not necessarily an absence of communication or of communicative features. However, when communicative action is distorted in the above examples, it becomes difficult for me as a researcher to interpret the interactions as communicative action. Where is the rationally motivated consensus that guides group action towards some taken as shared network of statements and objects? Also the more traditional issue of confusing the teacher's behavioural authority with her expertise in the discipline and thus getting the wrong idea about the structure of mathematical knowledge (i.e. that it rests upon external authority and not the better argument) is potentially symptomatic of systematic distortion of the meaning of the discipline. The analysis in this chapter seeks to indicate the potential productivity of some of the analytical statements developed in Chapter 6, and thus the productivity of the codes and models developed in Chapter 5 in line with the integrated strategy for analysis (see Section 3.2.2).

Insights into how distorted communication can express itself in classroom activity can support the development of strategies and practices that develop understanding through communication in mathematics classrooms. In the next chapter, features of classroom practice and pedagogy that relate to the establishment of something like an ideal speech situation are examined and implications are drawn for how this may relate to creating the conditions for communicative understanding in a classroom setting.

Chapter 9: The potential for approaching an ideal speech situation

In this chapter two main issues are addressed. First, particular episodes of utterances that feature teachers' actions are analysed as supporting communicative action. Following on from this, normative features of complex instruction are analyzed from the perspective of the theory of communicative action and related to dynamic ideas of agency. My interpretation of relevant data suggests that establishing equitable norms may be a necessary precondition for the establishment of intersubjective understanding. Finally, there is a discussion of how this work could serve as a basis for more sociologically informed future work that moves beyond the consideration of micro-analytic features and begins to address wider social questions.

In Chapter 8 some of the ways in which communicative action can break down and thus fail to follow through on the promise to serve as a means for students to learn were examined. In this chapter the analysis focuses on what is being done by students and teachers working under the constraints and affordances of a complex instruction approach to meet the challenge of learning collaboratively. This analysis, proceeding from a point of view of communicative action, is based on a consideration of the notion of the ideal speech situation and how the complex instruction style of teaching, employed by the teachers in this study, might be interpreted as creating conditions that approach and promote the features of such an ideal situation. This discussion of features of communicative action will be complemented by a consideration of ideas about agency and practice in mathematics education and collaborative learning.

The data examined in this chapter serve to illustrate some of the actions undertaken by students and teachers in the course of interacting that can be interpreted as supporting communicative action. Analysis of these episodes of utterances may shed light onto the technical potential of complex instruction to foster learning in small group situations and inform how these approaches may be built on to foster more effective and equitable learning and teaching.

9.1 Examining evidence of an intersection of agency

Beyond the norms, roles and skill-builders (see Section 2.4.2), what characterised the interactions in the data in this study was the way in which teachers engaged students communicatively. By setting up situations where students could engage their agency through the rationality of communicative action around rich mathematical tasks, the teachers created situations where they were able to act as participants in students' interactions and model contributions that were focused on aspects of communicative rationality that were particularly important for learning mathematics.

The following excerpts from episodes of utterances highlight the moves that teachers made to engage and intervene in the communicative interactions of small groups. What is notable in these examples is that the teachers are interacting with students as participants in discursive communication. They rarely act to use their authority to determine or correct the constative[28] content (see Section 6.2) of the students' interactions. That is to say that the teachers do not address directly the truth value of the propositional content of utterances which are oriented to the mathematical task by 'telling the answer' or asserting evaluations of such utterances. Rather they ask probing questions, guide the students to coordinate their own interactions around the problem, and generally seek to give the students space to 'think for themselves'. This is a risky endeavour for them in some ways as the teachers in this study remarked several times that they were not sure that the students were 'getting what they were supposed to get' out of the lessons. But what were the students getting? It is possible that the modelling of practices of validity-discourse are important in themselves as they may give access to participation in practices that constitute understanding. It should be noted that this is not always necessarily successful, as my analysis (Section 8.2) indicates. The teachers are attempting to manage the tension between letting the students problem solve and explore on their own, and making sure that the students have access to the learning trajectories that were planned in the development of the lessons.

The next excerpt has been seen before in Sections 6.2 and 7.4. It is an example of the researcher acting in a participant observer role in one of the lessons. The researcher intervenes in a small group interaction and attempts to maintain a stance towards the mathematical knowledge that is focused on justification, rather than letting the students

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[28] i.e. utterances with propositional content that can be judged true or false such as an assertion or the relating of information.

take his comment as merely an evaluation of the truth content of the students' work. This focus on justification can be interpreted as indicative of an attempt to maintain conditions for 'validity-discourse'.

Harry: ok maybe 2 but [Conjecture: Identification of mathematical property, tentative; SPS]

Thomas: what prime numbers have you got [Question: orientation towards others' understanding; QPS]

Harry: 3…5….7 [Question: orientation towards others' understanding; RPS]

T[leaning over and interrupting]: is two a factor of three? [Teacher intervention: validity challenge; QD]

Students: oh- no [Charlotte starts erasing][Response to validity challenge; RD]

Teacher: wait-wait-wait, just answer the question: is it? [Teacher intervention: maintaining discourse- restating question; TD]

Students: no [Response to question; RD, RPS]

Teacher: ok How do you know? [Response to question, probing question; TD, QD]

Thomas: because you can't double it to make three and you can't [Response to probing question; articulation of reason; RD]

Harry: so prime numbers only have two factors… [Identification of mathematical property; SPS]

Thomas:– I don't know a way to uh….

Daniel: and there's nothing you can times it by to make…. [Broadening reason to more general justification; SPS]

Thomas: that's not entirely true… [Statement: Validity challenge; SVCh]

Teacher: there's no whole number [to Daniel] [Teacher intervention: clarification; TD]

Figure 23 Excerpt 9.1 Transcript 22062009GCMPFACTORSFP6

This is an example of the researcher attempting to deflect their authority to that of the discipline by engaging the students discursively. The researcher challenges a claim that the students have made and then instead of letting the student (almost reflexively) change the claim unreflectively, prompts the student to identify the justification for the change in the claim, which the student does by recourse to mathematical knowledge. Several different students respond in quick succession, one student Harry responding with an idea that is not directly pertinent to what the teacher is saying, while another Daniel tries to broaden a reason provided by the student initially engaged by the teacher (Thomas). Thomas challenges this statement on the grounds that it was not precise enough. The teacher steps in at this point and conditionalises the statement by Daniel in response to Thomas's validity challenge. In this situation the researcher is participating in the communication and discourse of the group around a question. However he is also was the one who brought the question to the students and insisted that they provide reasons for their answers. The researcher is using his authority to shape the conversation and is adding to the constative content of the discussion.

However, the researcher is also engaging the students as a participant in an episode of communication where he is trying to engage the students as a relative equal before the discipline (although only equal in the peculiar sense of wanting to locate justification within the discourse of mathematics). He could have merely said 'Two is not a factor of three'- that is what the students seemed to assume to be the intent of the question at first. By stopping the students from rushing to correct their work and instead focusing on the conversation on justification he is shaping the relation between his authority as a teacher and the authority of the discipline of mathematics. It is not true because he says so; it is true because there is a mathematical reason. In fact he could be said to be modelling key elements of communication that are necessary for understanding. He has asked a probing question, challenging the validity of a claim made by one or more of the students, and has insisted that the students provide reasons for their answer, and responded to a validity challenge by another student by suggesting an adaptation to Daniel's reasoning in order to maintain consensus and respect the contributions of Thomas and Daniel. Now there are assumptions being made by the teacher as well, and it could be that the constative nature of the intervention at the end of the episode was a missed learning opportunity to inquire as to the detail of the Thomas's objection to Daniel's reasoning and then coach them in a conversation and allow them to reach consensus more on their own. The researcher has no direct evidence that all the students in the group are clear on his contribution.

In the next excerpt, Ms. Phelps returns to the same group later in the class and engages with them around some of the ideas that they have been developing. Ms. Phelps engages in some expectation management with an eye towards a learning trajectory focused on the foundational parts of the task, rather than on the extension.

Ms. Phelps: What questions are you going to ask yourselves what questions are you going to find out about and what have you decided to do…. Just quick then it doesn't have to be full sentences just share some things….so what have you decided to do? [Teacher intervention: implicit activity orientation; TC, TD]

Charlotte: We just sat down and we've given the jobs the two questions… so they're doing the um patterns one between three four five factors and we're doing [Response to teacher intervention: description of (attempted)division of labor; RD]

Thomas: we're doing the thousand… [RC]

Harry: we're doing the million…. [RC]

Ms. Phelps: ok… do you think they might be related...? [Response to teacher intervention: description of (attempted)division of labor; TD, QD, TC]

Harry: uuum…….yeah…. [Response to teacher intervention: agreement, hesitant; RD]

Ms. Phelps ok- How are you going to about finding how many factors a million has? [Teacher intervention: probing question; TD]

Harry: well we looked at a hundred and there's….well a hundred's got nine factors and we... [Response to teacher intervention: articulation of problem solving; RD]

Ms. Phelps: I just….a million's quite challenging… I just wondered if you need the work you've allocated to these guys in order to help you – you need to know something about the patterns because what you're trying to do is not count them, so I think it's a bit ambitious to answer the second one without answering the first one[Teacher Intervention: suggestion orientation on inclusion, justification based in mathematical nature of tasks; TC, TD]

Thomas: shall we all work on that...? [Response to teacher intervention: orientation to implied directions; RD, RC]

Figure 24 Excerpt 9.2 Transcript 22062009GCMPFACTORSFP6

Excerpt 9.2 is indicative of the balance that the participants in this study are trying to achieve between wanting the students to work independently (as groups) and wanting to guide them towards the learning trajectories that they have imagined in planning the tasks and lessons. In Excerpt 9.2, above, the teacher is engaging as a participant in the small group interaction and modelling discursive practices and emphasizing groupwork norms. She is also communicating pacing norms and expectations of task completion and this dual role again indicates the challenge of promoting equitable norms for participation in the context of the power relations of a classroom.

Ms. Phelps: that's actually not a bad idea and then as soon as you find some patterns and you're like ooh now I know what we need to do you can split off maybe but I think that the key because you haven't got that long is to think about finding some patterns and then choosing one thing that you think is really exciting and different to share with the rest of the class. [Teacher intervention: affirmation of response, orientation on collaborative problem solving norms; TC][Teacher leaves table]

Harry: so…

Thomas: do we need to find a pattern? [Question: clarification/probing; QC, QPS]

Harry: yeah we'll just work on this one, as soon as we get all a hundred done, which we should be able to do in fourteen minutes... [Statement: orientation working together on task; RC, RPS]

Charlotte: one two … three doesn't go into fourteen does it? [Statement: orientation identifying factors; SPS, QPS]

Harry: fourteen is almost a prime number …. Four doesn't go into fourteen…it's one two… seven and fourteen [Statement: random assertion; Statement: orientation identifying factors; SPS]

Figure 25 Excerpt 9.3 Transcript 22062009GCMPFACTORSFP6

In the next section (Excerpt 9.3) Ms. Phelps engages with the same group near the end of the class. She attempts to delegate authority for reasoning to the students by redirecting them to each other as resources rather than depending on her for constative content (discrete answers). This would seem to be a good strategy, however it is not clear from the next excerpt that the students engage with Ms. Phelps' move beyond getting the message 'you should know that'. The group (or at least Harry and Thomas) returns to pursuing the extension portion of the task that Ms. Phelps had previously tried to steer them away from.

In this intervention (excerpt 9.4, below) the probing question from the teacher complements the explicit statement by orienting the students towards the possibility that they should be able to determine mathematical truths for themselves without recourse to the teacher. However, while the teacher clearly meant for the students to think about how they would know for themselves whether or not they have found all the factors, there is no evidence in the transcript that the students have engaged with this question as Harry and Thomas rush back into groping for a pattern that will get them to the number of factors that one million has.

Charlotte: two times what gives eighteen? [Question: orientation identifying factors; QPS]

Ms. Phelps: you guys don't have to ask me about factors you can check with each other- how do you know if you've found all the factors? [Teacher intervention: deferring authority for mathematical content back to students; Probing question; TD]

Daniel: we've only got through a quarter of this in…..twenty minutes….we've done a quarter of this in twenty minutes [Statement: orientation time constraints; SC]

Thomas: it doesn't matter no we don't want to know that Daniel [Response: denigrating Daniel's pacing issue; RC, RD]

Thomas: so we've asked each other: what do you multiply by itself to get a million? [Statement: orientation interactional content; SPS, SD]

Harry: square numbers [Response: orientation to the question repeated; RPS]

Thomas: those aren't square numbers [Statement: validity challenge (tacit); SVCh, RPS]

Harry: a thousand…. What's the relationship between ten and a hundred? A hundred's got nine ten's got four…um….it's basically doubled but…. [Statement, Rhetorical question, statement: orientation on trying to guess a rule; SPS]

Thomas: doubled add one….so – a hundred to a thousand would be eight- nineteen [Trying to build on (somewhat incoherent) guesses about how to find the number of factors in a million by working up from 100 to 1000; RPS, SPS]

Figure 26 Excerpt 9.4 Transcript 22062009GCMPFACTORSFP6

This rush to generalise without establishing key concepts is reinforced by Daniel's invocation of the time constraint they are under and how they have completed comparatively little of the assigned task. This reinforces the analysis that there are conflicting norms and expectations at play in the classroom. On the one hand the teacher wants to support the agency of the students in tackling mathematical tasks, on the other hand the constraints of the patterns of instruction in classrooms and schools creates pressure on the students to act without necessarily reflecting (or having space/time to reflect) on the mathematical ideas they are working with.

To what extent can interactions like these be understood in light of the concept of the ideal speech situation? Can they be understood as communicative action at all? I would suggest that these patterns represent a constellation of strategic and communicative acts. These constellations of action exist in parallel and nested fashion, such that the teachers are trying to communicate and model communication while at the same time guiding investigation and controlling behaviours. This has important implications for understanding how approaching an ideal speech situation could be thought of in a classroom situation, and how this relates to the interaction of agency between the teachers, the students, the wider school culture and the discipline of mathematics.

Mrs. Boxer: can you tell me what you've discovered so far? [Question: orientation to students work; TC, TD]

Matthew: I think that the circumference is three times the width [Statement: articulation of conjecture; RPS]

Mrs. Boxer: Really? How did you work that out? [Question: orientation on justification; TD]

Matthew: because most of them are quite close [Response: articulation of justification; RD]

Joseph: Cuz he's looking at the pattern [Response: supporting justification; RD]

Mrs. Boxer: Why don't you jot that down so that you don't forget that thought? [Not really a question: orientation on coordination of student work; TC]

Matthew: Width is a third of the circumference [Statement: different articulation of conjectured relationship; SPS]

Mrs. Boxer: So if I wanted to um put a ring into a box that was two centimetres wide...? [Probing question; TD]

Matthew: six [Response based on conjectured mathematical property; RPS]

Mrs. Boxer: You'd have a bracelet that was six centimetres round [Follow up- adding to articulation of student's response; TD]

Matthew: Yeah [Response: affirmative; RD, RPS]

Mrs. Boxer: OK, So can you use that information to no work out how much the silversmith would need to make this bracelet? What does the bracelet look like any way? Does it look like this? [Holding up a card circle][Question: probing, extending, orientation towards constraints of task; TD, TC]

Joseph: But with a hole in the middle of it [Response: orientation constraints of task, properties of task objects; RPS]

Mrs. Boxer: With a hole in the middle of it. What do you need to know in order to...? [Response: affirmation; Question: orientation on constraints of the task; TD, TC]

Matthew: How thick he needs the edge [Response: identifying necessary (missing) information; RPS]

Mrs. Boxer: Well done [goes off to get the clue card with the relevant information on it][Response: evaluative; TC]

Figure 27 Excerpt 9.4 Transcript 10072009GVMBCIRCLESFP3

In the excerpt 9.4, we see another example of these overlapping goals in Mrs. Boxer's interrogation of the students' work and thinking. Her questions are probing at times, while in the next moment they are leading. Overall there is a tone of evaluation and direction. Yet patterns of communicative action are still interpreted as indicated by the coding.

Here the teacher is acknowledging the pattern that the students have found and then prompting them to consider how it was related to the goal of the task. The students responded with what they would need to know in order to move on, the information was provided and the investigation continued. This is an interesting example of how working towards the goal of the task can entail understanding. The students, in trying to complete the task have articulated an important (if nascent) geometric relationship.

Whereas in Chapter 7, excerpt 7.3, the same students were negotiating how to carry out an action (measuring), in this excerpt (9.4) it seemed more that one student was noticing a pattern and sharing it only when prompted by the teacher. The students have come to a partial understanding of what their goals are and have come up with some limited strategies for pursuing them. The actions that contributed to the information that Matthew seems to have considered in coming up with this insight were undertaken by Matthew, Callum, Joseph and Samuel and at least one of the others is engaged in the conversation above, which might be interpreted as evidence of communication, and thus potentially be based in a shared understanding about the mathematics in the task. However it is difficult to interpret the level or extent of mutual understanding around the task from the data.

In this final section (excerpt 9.5) the students from this group share the insight they had with the whole class. It is interesting to note the misunderstanding articulated by Callum (line 25), and how the teacher handles eliciting the articulation of the insight and its relation to the wider task. Beyond these examples of the teachers intervening in communication as an active participant there are also the norms that are set up in the classroom and the planning of the tasks that are conducive to the maintenance of effective groupwork. In this sense one might say that effective groupwork displays some features of an ideal situation and in which communication around a common problem is taking place.

Again this maintenance of conditions for effective groupwork is interpreted as the teachers engaging in a constellation of strategic and communicative practices that aim at creating a situation strategically where students may be able to realise communicative participation in learning and thus have access to the mutual understanding entailed in such communication.

Matthew[addressing whole class]: We've found that the width is a third of the circumference [Statement: articulating conjectured property; SPS]

Mrs. Boxer: Wow. how did you find that out? [Statement: expression of interest/affirmation; Question: Probing; TD, QD]

Matthew: Because when we did it we measured the width and then the circumference and they all came round to about a third like. one was 5.2 and the circumference was 15.9 [Response: orientation on justification; RD]

Mrs. Boxer: So that would definitely help the silversmith [Statement: follow-up, orientation task context; TD, TC]

Matthew: yeah and then we had another go to see if it was right and it was [Statement: orientation further justification; SPS, RD]

Mrs. Boxer: So what are you saying to me; if the silversmith knows... [Rephrasing insight, leading question; TC, TD]

Callum: the width [Response: trying to follow teacher's lead; RPS]

Matthew: Yeah if the box is [Response: trying to add to articulation; RPS]

Callum: The circumference is a third of the width [Statement: articulation of property (incorrect) in context of expressing insight to whole class audience; SPS]

Matthew: No. The circumference is times three of the width [Validity Challenge: correcting mis-statement of property; SVCh, RD]

Matthew: So if the box is ten centimeters wide the ring'll be thirty [Statement: articulating insight in the context of the task's story; SPS]

Mrs. Boxer: right so you've got some evidence you've tried with different circles for that one. very interesting, very interesting indeed.

discussion moves on to another group.

Figure 28 Excerpt 9.5 Transcript 10072009GVMBCIRCLESFP3

An important feature of the code schema developed in Chapter 5 Table 3 is that it can be interpreted as characterizing the different moves made by the participants in the interactions in this study. Teacher interventions are seen mainly in the coordinating and validity categories, with little or no teacher activity in the problem-solving (or constative) category. This suggests that the teachers are engaging students as participants in their small group interactions, and that while this is done from a position of power it is also primarily done in an attempt to support and guide the students' own communicative action towards completing the task. This type of scaffolding and modelling of communicative moves may be understood as strategic action intended to support development of communicative practices in the students.

This interpretation raises serious questions about understanding teaching and learning from the perspective of a communicative model of understanding. Ms. Phelps is acting strategically here to try to establish the more or less equitable relations of a communicative situation. Does it make sense to act strategically to establish conditions for communication (which is conceived by Habermas as fundamentally non-strategic)? What does this imply about the power-relations that exist within the classroom and school contexts? I suggest that these questions are key to understanding what the teachers are doing in this study: they are acting strategically to create the potential for communicative participation by their students. I suggest this is the essence of a constellation of practices employed by teachers in aiming to use equitable teaching approaches to teach mathematics with understanding.

9.2 The ideal speech situation, communicative action, and the potential for design

What evidence from the data would show evidence of the approximation of an ideal speech situation? Teachers' reflective comments and planning discussions (see Section 4.3) indicate that they wanted the students to think for themselves as opposed to slavish dependence on the authority of the teacher. Also the belief that a complex instruction approach, which explicitly emphasises these ideas of equitable participation, is being chosen by the teachers could be interpreted as indicative of a belief by some practitioners of the potential for such norms to exist, at least in some approximation (given the power laden reality of schools and classrooms).

In this section I shall trace some important features from Habermas' concept of the ideal speech situation and seek to explore the extent to which complex instruction (as imagined and as used) can be analysed as approaching such an ideal. There seems to be potential for contradiction between what might be characterised as the strategic actions of the teacher in trying to create more equitable classroom conditions and the egalitarian principles to which such an ideal speech situation imagines and aspires. While it can broadly be said that the ideal speech situation, referring to Habermasian discourse, is simply a set of principles asserting equity of un-coerced participation in that discourse, the issue deserves more detailed treatment because of the interesting details of Habermas' argument, and how these details relate directly to the interpretation of educational settings using complex instruction styles of pedagogy.

Habermas posits that a key feature of the consensus theory of truth is the mutual presupposition of an 'ideal speech situation'. This concept is necessary because Habermas denies the possibility of an external, independent arbiter of the competence of participants in deliberation. For Habermas this raises a dilemma, of why in the absence of independent assessment of competence of participants we can assume that we can reach mutual understanding and tell a rational consensus from an illusory one.

> I would argue that what explains it is that the participants in argumentation mutually presuppose something like an ideal speech situation. The defining feature of the ideal speech situation is that any consensus attainable under its conditions can count per se as rational consensus. *My thesis is that only the anticipation of an ideal speech situation […] warrants attaching to any consensus that is in fact attained the claim that it is a rational consensus.* At the same time, this anticipation is a critical standard that can also be used to call into question any factually attained consensus and to examine whether it is a sufficient indicator of real mutual understanding. (Habermas 2002, italics in the original)

This is a key argument for this thesis and allows insight into how and why one may usefully seek recourse to some of these ideas in the interpretation of the episodes of utterances in this study. It has to do with the way in which Habermas frames the outline of this speech situation. He asks the reader how we might design such an ideal speech situation, and then goes on to outline its features.

> How is it possible to design an ideal speech situation by means of speech acts that every competent speaker knows how to perform? In terms of distinguishing between a true and a false consensus, we call a speech situation ideal if communication is impeded neither by external contingent forces nor, more importantly, by constraints arising from the structure of communication itself. The ideal speech situation excludes systematic distortion of communication. Only then is the sole prevailing force the characteristic unforced force of the better argument, which allows assertions to be methodically verified in an expert manner and decisions about practical issues to be rationally motivated. (ibid.)

Habermas asserts that there must be an equal opportunity for all participants to choose and perform any speech acts. The important conclusion to this is the assertion that if such an ideal speech situation could be designed, it would feature an equal distribution of opportunities to employ 1) 'communicatives' such as speaking, responding, asking questions, and giving answers, and 2) 'constatives' such as interpretations, assertions, explanations, or justifications (Habermas 2002). This is pertinent for analyzing the extent to which complex instruction can be seen as a way of attempting to establish the conditions for meaningful communication. This argument is addressing directly design features of a situation in which communication, and particularly 'validity-discourse', could take place. There is one further aspect of the ideal speech situation that Habermas outlines, and that is that participants must deceive neither themselves nor others as to their intentions.

This leads to a complementary idealization of communicative action, which he refers to as a model of pure communicative action. He claims that these ideals are necessary for the possibility of communication. Though again he ascribes to them the status of counter-factual norms, which must be assumed even though they cannot be shown to be true.

> If we encounter an other as a subject and not as an opponent, let alone an object that we can manipulate, we (inevitably) take her to be accountable for her actions. We can only interact with her, or as I have put it, encounter her at the level of intersubjectivity, if we presuppose that under appropriate conditions she could account for her actions. Insofar as we want to relate to her as a subject, we must proceed on the assumption that the other could tell us why in a given situation she behaved as she did and not otherwise. (Habermas 2002)

This leads to an idealization that has two parts: 1) an expectation of intentional action; and 2) an expectation of legitimacy. That is to say that actors must be treated as though they are aware of the reasons they are doing what they are doing, and that they are only following those norms they take to be justified. This has a number of consequences for Habermas such as denying recourse to ascribing unconscious motivation to a participant without leaving the realm of intersubjectivity. This idea of leaving the realm of intersubjectivity indicates the potential for communicative action to devolve into strategic action, such that the participants in an interaction no longer accept each other's justifications on the basis that they do not accept the sincerity of the reasons articulated as justification for utterances (and other actions). The consequence of this is that mutual understanding is no longer necessary for coordination of action and thus the potential to build mathematical understanding on the basis of a shared understanding is jeopardised.

Thus Habermas' conception of an ideal speech situation and its relation to an ideal model of communication dictate conditions that could be thought of as design principles for ensuring meaningful intersubjective communication. The design principle's emphasis on equitable participation and constative and communicative acts are meaningful because they serve as a key part in an argument that issues of equity are not solely social justice issues, but may also be connected to technical processes of understanding that are alive in the classroom and which may serve as the basis for developing mathematical practices and knowledge in students. Simply put, certain aspects of equity can be seen as essential to the communication involved in effective teaching and learning. But how can these be achieved in the power-laden context of a classroom situation? I suggest that the constellation of practices in teachers' actions analysed in this thesis reflect an attempt by teachers to use their institutional power and their authoritative power to preserve (or create?) equitable relations that foster communicative understanding in the service of teaching mathematics.

9.3 Complex Instruction as an attempt to create conditions approaching an ideal speech situation

In this section the elements of complex instruction that can be interpreted as aimed at creating conditions that are similar to the ideal speech situation outlined above are highlighted and some important differences are noted. Reference to how these aspects of practice played out in the various classrooms in this study are discussed and then an argument is made that the actions of the teachers in this study can be interpreted as aiming at creating a set of constraints and affordances to facilitate students communicative engagement. Finally, the relationship between the inherent rationality of communication and the rationality of mathematical practices and knowledge is examined.

Complex instruction, as noted previously (see Section 2.4.2), is a set of teaching practices focused around designing effective conditions for learning using groupwork. Complex instruction is focused on increasing the participation of all students with the understanding that students learn more when they participate. In order to increase participation, complex instruction identifies status issues as particularly problematic. In Cohen (1994) status rankings are seen as hierarchies that tend to develop in small groups such that the participants see those with higher rankings as more competent. This status-ranking concept is somewhat ambiguous, because high rates of participation and high competency judgments are treated as coincident. There are four aspects of status that Cohen raises as potential issues in the design of groupwork; expert status, academic status, peer status, and societal status. Expert status and academic status are different in that where academic competency is perhaps seen as an appropriate measure of the value of the contributions of a participant in the first case, in the second case academic competency is not directly related to the value of the participants potential contributions (for instance a student good at reading dominating a role-play activity). In each of the cases the issue is that some students are participating more and their contributions are see as more valuable, while other students are participating less and their contributions are seen as less valuable.

Complex instruction seeks to address the status issues that arise commonly in the use of groupwork by delegating responsibility for various contributions to different participants, creating norms that encourage students to work collaboratively and using skill-builders to train students to collaborate productively. Combined with this are strategies called 'status-treatments' that seek to undermine negative status rankings such as valuing the contributions of students with low status. Through these techniques complex instruction seeks to promote more equal rates of participation. Further, the use of heterogeneous classrooms reduces the opportunities for students to see themselves as inherently unequal. The efficacy of these approaches has been documented in the research literature (Cohen and Lotan 1997).

In complex instruction style pedagogies promotion of equitable rates of participation is justified based on analysis of empirical evidence focused on the relation between rates of participation and learning outcomes, as well as arguments in favour of equitable access to education. I argue that the teachers in this study, who are adopting some elements of complex instruction style pedagogies, can be interpreted as creating the conditions that are conducive to meaningful communication from the point of view of Habermas' theory of communicative action.

9.4 Agency and communicative rationality

In this section I discuss two related issues. First, the relationship between a rationality inherent in communication and the practices and knowledge specific to mathematics; and second, the way in which teachers seek to facilitate the enculturation of mathematical knowledge in the context of communicative action. In the first instance the argument will suggest that everyday rationality serves as a necessary foundation for the development of mathematical knowledge, and in the second instance I will argue that there is evidence that can be interpreted as teachers attempting to support the development of mathematical knowledge and practices by engaging the rationality inherent in students' communication.

There is a connection between the everyday rationality of the lifeworld, which is constituted intersubjectively through communicative action, and the rationality of the practices and knowledge of the discipline of mathematics. Boero and Morselli (2009) discuss Habermasian rationality and proof, suggesting that the challenge consists of leading students from ordinary argumentative practices to the sophisticated and highly situated practices of proving. From the standpoint of a theory of communicative action, the situated practices of the discipline of mathematics are both framed by and constitutive of cultural institutions that depend for their dynamism on a rationality inherent in communicative action (Habermas 1985b).

What is key to understand in this is that communication allows participants to not only engage their own agency but it also requires participants to posit the agency of others. This is the idea of intentionality that has been mentioned before (see Section 2.1.6, and Section 3.1.1). In the course of acting, communicative participants must tacitly posit the intentionality of others, each in a reciprocal fashion. This is the basis of intersubjectivity and is part of the potential usefulness of a communicative and intersubjective perspective.

In her studies of teacher practice, Boaler posits a 'dance of agency' between the student, the teacher, and the agency of the discipline of mathematics as a key feature of the practices that allow some teachers to achieve dynamic learning results from their students (Boaler 2003). In her articulation of this dance she describes how one teacher taught in a way that used open problem-solving by positioning the discipline of mathematics as the authority from which students should draw insight into their problem-solving, while at the same time valuing the students' own contributions.

> In Ms Conceptual's class we frequently witnessed students engaged in this 'dance' – they were not only required to use their own ideas as in Mr Freedom's class nor did they spend the majority of their time 'surrendering to the agency of the discipline', as in Mr Life's class; instead they learned to interweave standard methods and procedures with their own thoughts as they adapted and connected different methods. (Boaler 2003)

In my research, the analysis suggests that there are productive and non-productive episodes of teacher intervention. In some cases there is evidence that can be interpreted as indicating that the moves the teachers are making are similar to the kind of complex relationship with the discipline Boaler describes. However, in other episodes teachers' actions seem much more about directing students towards a preconceived learning trajectory. At times, as the students are engaged in problem-solving, teachers observe and intervene in two broad categories. One way is to coordinate the students to focus on the task and the collaboration required to complete it, the other is to intervene communicatively. This is where ideas of agency, intentionality and communicative action can help us gain a technical insight into what the teachers are doing. The analysis of episodes in this thesis suggests that this constellation of strategies employed by the teachers in this study does not always achieve the kind of fluidity described in Boaler's work. However, some of the elements are there and the analysis in this thesis seeks to illuminate the tension between the strategic and communicative elements in the constellation of practices employed by the teachers.

9.5 Connecting macro and micro levels of analysis: the potential of this analysis to inform future work

The research findings of this thesis provide a basis for a more sociologically informed analysis to address social inequalities in future work. Habermas' ideas of system and lifeworld (Habermas 1985b) could be expanded in the first instance, building on the discussions of steering media and the relationship between the strategic and instrumental action of large scale systems in society (such as politics and the economy) and the communicative action that is meant to characterise the cultural practices of the lifeword. The research and findings in this research are essentially describing micro features of the lifeworld of particular mathematics classrooms. This approach could be broadened first to take into account more detail of the lifeworld in which the small group interactions are situated. An ethnographic approach could be used to analyse the socio-cultural features of the lifeworld in which the small group analysis is situated and then survey methods and anthropological approaches could be used to address the systematic features which exist at more macro levels of the institutional and social context of these classroom based analyses.

Alongside fleshing out the analysis of the macro/micro features from a Habermasian perspective, a broader analysis (perhaps as a discussion between different analytical perspectives) could be undertaken, considering multiple theoretical resources and research based in other theoretical traditions. The work in this thesis can be seen as an indication of where to turn our gaze as researchers in order to understand particularities of social inequity in mathematics teaching and learning and one way to engage with them at the micro level. Working backwards from there one could explore the resistances (at a more macro level) to mitigating the kinds of inequity and communicative pathology found at the local level. Decisions about how to approach the problem and whether (and how) to attempt to draw upon a wider range of theoretical resources could be made prior to the attempt to address these empirically. In particular these ideas could influence the research design and plans for analysis. Theoretical resources from Bernstein, Bourdieu, and Foucault could be used to complement the systems theoretical critique based in Habermas (see discussion in chapter 2 section 2.6).

Future work might incorporate anthropological and ethnographic analysis of lifeworldy practices (complementing this research) done in conjunction with sociological surveys of institutional cultures and structures could be approached analytically using constellations of critical social theory and drawing from existing empirical research to address the ongoing particularities of the persistent inequitable status quo in mathematics teaching and learning. The possible fruitfulness of this approach, and also the challenges posed by such an undertaking are indicated in Bernstein (2000) in his discussion of horizontal and vertical discourses:

> This analysis will proceed by distinguishing two fundamental forms of discourse which have been subject to much comparison and contrast. The two forms are generally seen as oppositional rather than complementary. Indeed one form is often seen as the destruction of the other. Sometimes one form is seen as essentially a written form and the other is essentially an oral form. Bourdieu refers to these forms in terms of the function to which they give rise, one form creating symbolic, the other practical mastery. Habermas sees one form as constructing what he calls the 'life world' of the individual and the other as the source of instrumental rationality. Giddens following Habermas sees one discursive form as the basis for what he calls 'expert systems'. These 'expert systems' lead to a disembedding of individuals from the local experiential world which is constructed by a different

> form. Underlying these contrasts or oppositions is a complex multi-layered structure of pairs operating at different levels of individual and social experience. (ibid. pp155-156)

This integrative analysis from Bernstein illuminates some of the potential connections between a number of different theorists attending to the issues of how macro and micro features of social reality relate to one another. However, building on the potential for such analysis would require a major effort of networking theoretical perspectives, considering their congruence and their divergences and the ways in which these might inform analysis in a complementary fashion. While the research done in this thesis was approached with this in mind (see Chapter 2 Section 2.7), the wider analysis is beyond the scope of this work, which confines itself to an exploration of Habermas' theories in the analysis of small group interactions in unset year 7 mathematics classes adopting aspects of complex instruction in England.

My initial analytical ambitions were, at least in part, to develop insights from micro-analysis such that the theories developed could serve as a basis for exploring connections between macro and micro features of communication in small group work in school mathematics settings. The primary focus on small group interactions revealed conflict and power relations which pointed towards a need to engage with a wider set of data, including details of contextual aspects of participants lives and identities outside the mathematics classroom. More detailed longitudinal data of the participants' mathematics classroom experiences would have allowed for analysis that connected meaningful interpretation of micro-analytical features in different settings in order to more fully understand small group interactions in school mathematics classes. These limitations should spur further reflection in research design in future work that sets out to explore the findings of this thesis. In the end, while this work did not secure findings about the connections between micro and macro features of the social reality of school mathematics, it did develop micro-analytic findings in a manner such that these issues could be productively explored in further, perhaps more sociologically informed, work.

9.6 Conclusion

The important implication regarding equity has to do with the fact that this system of meaning making is premised on an abductively arrived at ideal speech situation. This ideal speech situation entails counterfactual norms that act as preconditions for the achievement of intersubjective understanding. The ideal speech situation comes into play in discourse when consensus has been challenged, or has not yet been achieved, and must be established or re-established. The problem giving rise to the necessity of the ideal speech situation (which is conceived of as a counter-factual norm) is that criterion upon which to judge the competence of interlocutors can only be arrived at, under a consensus theory of truth, through the very consensus-process of which it would act as the foundation. This circle could only be broken by ontological theories of truth, which, Habermas claims, have not yet held up under scrutiny. And yet communication and understanding exist, which leads Habermas to the abductive step of outlining the ideal speech situation as a model for the kinds of conditions, which would have to exist in order for this state of affairs to be as it is.

This ideal speech situation, which is considered a precondition for the possibility of achieving intersubjective understanding, consists of 2 parts; 1) That no power is brought to bear in the formation of a consensus beyond the 'unforced force of the better argument'; 2) that all participants have symmetrical opportunities to participate in the discourse (Habermas et al. 2002). This can be broken down further- adding detail that any actor with the competence to speak and act be allowed to; that any assertion can be made; that any assertion can be questioned; and that actors be allowed to express their opinions, wants, and needs.

It is here that one can see the potential role of equity in establishing conditions for understanding. Moreover, one can see it in terms of practices of communication. In order for intersubjective understanding to be established, participants must act as though some form of ideal speech situation exists, otherwise the mutual understanding being developed will be undermined, consensus may fall apart or become distorted by power-relations resulting in flawed understanding or strategic action as opposed to communicative action.

Having established this, I now reflect on the normative aspects of Complex Instruction and suggest that the constellations of strategic and communicative actions that teachers employ in attempting to adopt these practices can be seen as aiming to establish norms of equity that mirror in important ways the counterfactual norms of an ideal speech situation. This can be seen in the ideas of skill-builders and the classroom norms articulated in Cohen (1994), as well as (to a lesser extent perhaps) the normative content of the roles.

This analysis serves as the basis for an important working hypothesis: To a certain extent the success (or lack thereof) of groupwork, in facilitating mathematics learning through discursive problem-solving, can be attributed to the extent to which equitable norms and practices are established in the classroom. This is not novel (Boaler & Staples 2008), except that the foundation for this claim is a technical one having to do with specific practices of establishing intersubjective understanding from a perspective of communicative action. Not only may understanding be distorted in the absence of such equitable norms, but students will also be denied access to models of practice that entail intersubjective understanding.

Chapter 10 Empirical and theoretical findings and implications for research and practice

The Thesis developed from the preceding analyses is that small group interactions in mixed ability year seven mathematics classes in England, where complex instruction style approaches are being used, can be understood from a theoretical perspective based in the Theory of Communicative Action. Further, it is argued that there is a critical potential to understanding interactions in this way. This thesis has sought to address the research questions: 1) How can we understand student interactions in the context of small group problem-solving in mixed ability year 7 mathematics classes adopting elements of Complex Instruction?; 2) Are there patterns of interaction, both among students in small groups and between students and the teachers? If so, what are their characteristics?; and 3) What is the critical potential of understanding student interactions from a perspective of communicative action?

The research underpinning this thesis set out to understand small group interactions in mixed ability Year 7 mathematics classes using elements of a complex instruction style approach. The use of a theory seeking case study approach, informed by Bassey (1999), gave structure to my investigations of episodes of utterances situated in the practices and cultures of mathematics classes and schools in England. The consideration of methodologies of participation and critical theory led me to adopt a stance based in Habermas' (1985) ideas of communicative action. These epistemological and ontological stances regarding the nature and scope of the social sciences grew alongside an analysis of episodes of utterances as meaningful intersubjective speech acts.

While the first two questions could have potentially been approached from established lines of group-dynamics theory and social psychology (or a number of other perspectives), the use of a more open approach to initial analysis opened up the way for an understanding based in ideas of intersubjectivity and critical theory that are in fundamental ways at odds with the tradition of ego-psychology developed in the US in the post-war period. Chapter 1 sought to locate the motivation for this research in a sketch of the author's identity as a teacher and prevailing (and conflicting) narratives of crises in mathematics education. Chapter 2 sought to review relevant literature while addressing with rigor the potential and limitations of networking theories in mathematics education. Chapter 3 sought to establish a methodological position that was coherent, flexible and rigorous, and to situate the practical methods of research employed in this thesis therein. In Chapter 4 the analysis of episodes of utterances was situated in the context of the collaborative development of tasks, the observed lessons, and the reflective interviews with teachers. Chapters 5, and 6 answer the first research question by developing codes and models that articulate an understanding of student interaction in the context of small group problem-solving. In Chapter 5, the practical methods of Bassey's case study approach were then employed through the integrated approach to analysis to develop codes and models, which served as first level analytical statements that were then taken back to the data in an iterative manner. Chapter 6 demonstrated how these codes and models were used to analyse episodes of utterances to generate 'higher level' analytical statements addressing patterns in the episodes. Chapters 7, 8, and 9 answer the second research question using the codes, model and framework developed to explore patterns of interactions in small group problem-solving. Chapter 7 continued the iterative process by analysing claims about the suitability of the theory of communicative action as a model for understanding the episodes. Chapter 8 further continued the iterative process in a parallel fashion by analysing claims about breakdowns and distortion of communication. Chapter 9 analysed teachers' actions as constellations of strategic and communicative action aimed at creating the potential for students to engage communicatively in mathematical learning. Chapter 9 also began to address the critical potential of understanding interactions from the point of view of Communicative Action by addressing the concept of the ideal speech situation and the role that it can play in the interpretation of small group interactions. Chapter 10 Section 10.2 answers the third research question by arguing that the analytical and theoretical work in this thesis has a critical potential, and articulates what this critical potential may be.

This iterative process of analysis generated theories, codes, models, and insights that taken together address the research questions in this thesis. The answers are articulated in this chapter as claims to knowledge, which have been generated through the analytical case study approach to the analysis of small group interactions in mixed ability year seven mathematics classes adopting elements of complex instruction in England.

During the course of this project, I refined the gaze of my research: I wanted to understand the small group interactions from a perspective of communicative action. Insights from initial open coding and the constant comparison of analysis of data led me to reflect that the conceptual resources I was employing in interpretation of the data were heavily influenced theoretical resources I was bringing to the analysis from Habermas. Rather than abandon this analysis as tainted by researcher bias (as one might from a more positivistic stance), I embraced it and saw the alignment of methodology and theory as fortuitous and potentially elegant.

However, simply adopting ideas and approaches from a perspective of critical theory does not make a theory critical, and there are thus two main sets of claims to knowledge that must be addressed in reflecting on the work presented in this thesis. First, there is a theoretical description of situated small group interactions based in empirical findings, which were refined through the iterative analyses following Bassey's (1999) case study approach. This set of claims is represented by the empirically focused analysis of the thesis from Chapter 4 onwards, beginning in Chapter 4 with a description of the participating teachers, their classrooms and practice and moving quickly in Chapter 5 to a fine grained micro-analysis of (primarily) student interactions in small groups. The codes and categories developed can be thought of as first stage analytical statements which were taken back to the data items, tested against them and refined through the course of Chapters 5 and 6 as an intersubjective model of small group interaction was developed. Chapters seven, eight and nine reflect another level of analysis as the model (which can be considered a constellation of analytical statements) was taken back to the data items, tested against them and refined. In Section 10.1 of this chapter I present the refined model and clarify some concepts, which I think are important theoretical pieces of this emerging description of small group interaction and how it may be understood.

The second set of claims to knowledge has to do with the critical potential of this work. From the very beginning I was concerned with not merely producing a description of a status quo that did little to address the potential for overcoming the potential for stagnation and destructive social conflict within modernity. I was, however, somewhat unclear on what such a critical potential entailed (as noted in Chapter 3 discussion of Habermas' ideas about understanding meaning in the social sciences). What is the normative basis for critique? What is the critical potential of understanding? In Section 10.2 of this final chapter I draw on theoretical resources in mathematics education (Skovsmose 1994; Boaler 2008) to articulate an understanding of the potential for critical theory to provide analytical insights that move beyond mere description of the properties of a case. This argument aims to realise the potential for 'bursting a given context open from within' and 'systematically exploiting the potential for critique located within the structures of communication against the particularity of the contexts of everyday action' (see Section

3.1.4). I consider Skovsmose's (1994) conceptualization of critique as it applies to this research, showing how the analysis of small group interactions developed in the research of this thesis reveals a potential for critique in the use of groupwork in the teaching of mathematics.

Finally, I review the main methodological, theoretical and practical contributions of this research. These constitute a third (overlapping and networked) set of claims to knowledge. I will reiterate the thesis that small group interactions can be understood from a perspective of communicative action and that this understanding entails a potential for critique of the use of groupwork in the teaching and learning of mathematics. I point out some limitations of this work and point to how this work might be developed further or inform other related work.

10.1 A theory of small group interactions

The first theme of analysis in this thesis addresses the first two research questions by articulating an understanding of small group interactions from a perspective of communicative action. This development of a theoretical framework and tools for analysis was achieved through the use of open coding, constant comparison, categorical refinement and integration, and iterative analysis of the data (particularly transcript data). The end result of this analytical development of theory is a set of codes for the interpretation of utterances, a model that relates these codes to one another in a dynamic fashion and indicates other important theoretical domains, and examples of analysis using these codes that reveals how communication can be interpreted as present in small group interactions, how it can fail or be partially achieved and how it may be supported by participant interaction. I will provide an overview of the elements of this theory in this section, discuss the refined model of student interaction (figure 10.1), and clarify some concepts that may help to flesh out the theoretical context and connections of the theory.

In Chapter 5 Section 5.4.1, I articulate the initial stages of theoretical integration with regards to the codes developed out of the initial stages of open coding. This integration led to an overlap in theoretical properties of codes. The schema developed to address this overlap was that utterances had two sets of functions that were being interpreted. The first was a grammatical function: utterances in small group interactions were typically questions, statements, and/or responses. The second set of functions was a communicative function:

the utterances were also interpreted as emphasising themes of validity discourse, problem-solving (and/or constative), and coordination of action. It was with these interpretations that I began to realise that the explicit use of theoretical ideas from communicative action was important (as I was evidently already using the ideas implicitly to make sense of the data). It is important to note that from a perspective of communicative action utterances in communication are not concerned with validity discourse, constative statements about reality, or the coordination of action by reference to implicit or explicit background norms. Rather utterances are conceived as always simultaneously making validity claims within domains of constative truth, sincerity, and normative correctness. However these various aspects are not all necessarily brought to the fore thematically in any particular utterance. Therefore what the interpretative schema developed in Chapter 5 indicates is the way in which various communicative functions of utterances were perceived as being brought to the fore by the participants in the small group interactions.

Some of the codes that survived the first rounds of integration were less useful than others in the subsequent analysis of data items. Thus the characterization of some utterances as, for instance, Action (AA: Active Action, AP: Passive Action, TA: Teacher Action, AD: Discursive Action) ended up being superfluous for analysis due to the fact that from a perspective of communicative action all meaningful utterances in a communicative interaction are speech acts and thus already a form of action. Thus the final Schema for Utterance Codes (Table 5) that ended up being used became more closely aligned with the theoretical framework of communicative action.

Other issues became problematic as the codes were used in the further analysis of transcript data. For instance the category 'problem-solving' raised multiple analytical issues. I was trying to characterise utterances that were focused on the mathematical tasks that participants were (ostensibly) working on. However, problem-solving is a concept that is already treated in many different ways in a variety of perspectives. In my analysis, if problem-solving existed in a communicative small group context it would be spread out over the entire model in a dynamic fashion (although not necessarily with equal rates of participation). What was the category I was trying delimit? I finally settled on constative, because the utterances that were being coded with these 'problem-solving' codes were addressing propositional content of statements oriented on mathematical objects (tasks or elements of tasks). In this sense they were constative utterances with propositional content that could be judge true or false in relation to the objects of communication. Thus the utterances coded as problem-solving were more correctly 'task-oriented constative statements/questions/responses' which did not emphasise coordination of action or validity-discourse. This is a much more limited notion of 'problem-solving', which might be better thought of as vernacular description of students working on a school mathematics tasks.

Another important adjustment to the coding schema was the clarification of discursive and validity codes. The concept of discourse and validity are used in a particularly technical fashion by Habermas (1985) in his development of a Theory of Communicative Action. In the TCA, Habermas defines discourse as the process of re-negotiation of consensus around validity claims in his three 'worlds' (subjective/objective/social). This is a very different use of 'validity' and 'discourse' than other traditions in the Social Sciences, especially socio-cultural theory which terms the wider narrative of various groups and sub-groups as particular 'discourses'. Thus there is a discourse of mathematics educators, and a discourse of policy makers, which can be characterised and potentially related to one another. Habermas would not disagree that such cultural narratives exist, but he uses the terminology differently and this can potentially cause confusion. Because of this, I decided to re-phrase the category of discourse as validity-discourse. Validity-discourse as a category is meant to refer to utterances that emphasise the validity dimensions of utterances in a communicative interaction as understood from a framework of communicative action. It is meant to distinguish the category from other uses of validity and discourse, and to highlight the technical role such utterances and interactions play in communication.

Table 5 Revised Utterance Codes, Schema and Key

Communicative Utterance Codes			
	Statements	Questions	Responses
Validity-Discourse	SV, SVCh, TD	QD, TD	RD
Constative and/or Problem-solving	SPS	QPS	RPS
Coordinating	SC, TC	QC, TC	RC

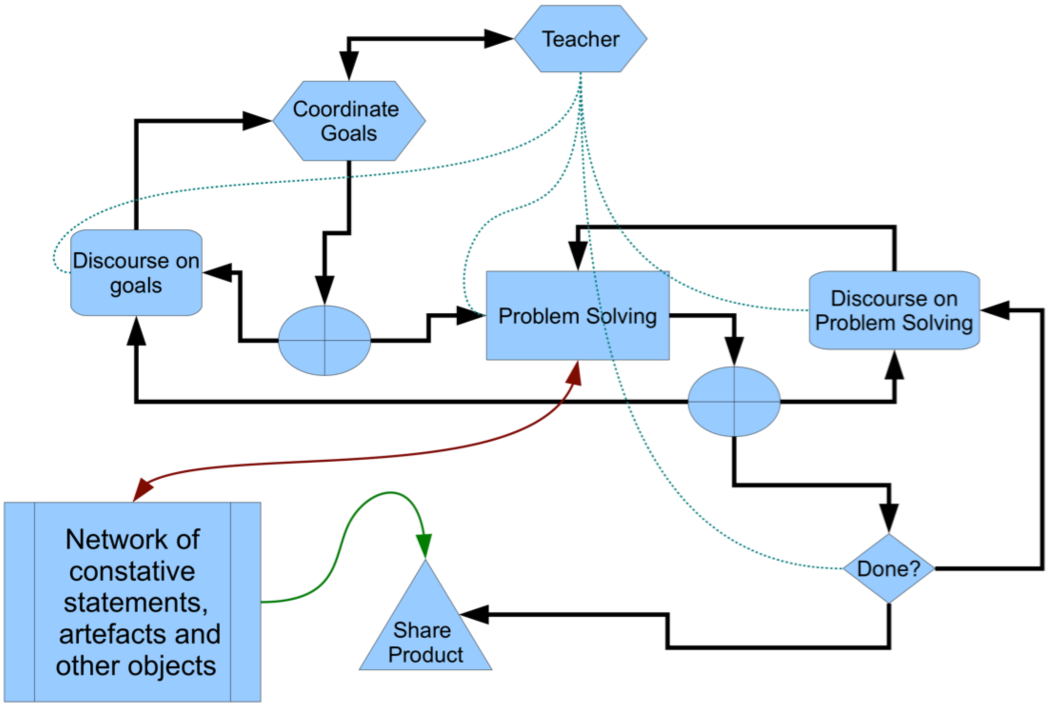
KEY:

Code	Description
SV	Validity Statement
SVCh	Validity Challenge
TD	Teacher Validity- Discourse Modelling
QD	Validity-Discourse Question
RD	Validity-Discourse Response
SPS	Problem-solving (constative) Statement
QPS	Problem-solving (constative) Question
RPS	Problem-solving (constative) Response
SC	Coordinating Statement
TC	Coordinating Teacher Move
QC	Coordinating Question
RC	Coordinating Response

The coding scheme is closely tied to an 'Intersubjective Model of Small Group Interaction'. This model, originally outlined in Chapter 5, Section 5.4.2, has been refined through the process of reflection during the iterative stages of analysis. The current version can be seen in Figure 30 below.

The teacher plays a crucial role, able to intervene at every stage (potentially), and is also responsible for setting the task to be considered, establishing and/or reinforcing normative expectations, and (potentially) for modelling communicative and mathematical practices. This is indicated by the prominence that the 'Teacher' has at the head of the model and also the dotted lines connecting the teacher to each stage of the model. However the rest of the model is focused on participants (which can include the teacher) interacting in small groups around a mathematical task. The model seeks to articulate patterns of interactive utterances in small group interactions. Students begin by coordinating their action by emphasizing the establishment of shared goals. These initial interactions, as the students try to understand the task, what the expectations are, and what the mathematical (and potentially other) content is, leads to either negotiation around goals (validity-discourse on goals), or to a relatively stable agreement about what they think they are supposed to do. In the latter case the students move onto 'working' on the tasks at hand. In this phase the students articulate ideas, opinions, and assertions of fact that contribute to a shared network of ideas that are oriented on the mathematical task and satisfying the goals related to the mathematical task.

Figure 29 Revised Intersubjective Model of Small Group Interaction



During this phase of groupwork four things can happen: The students continue making constative statements that contribute to the constellation of ideas that is the 'product' of their coordinated speech acts; the students' consensus around constative utterances break down and need to be re-established through recourse to justification in validity-discourse around problem-solving; the students' consensus around goals breaks down and has to be re-established through justification based on normative expectations such that consensus around the coordination of action is re-established and the students move back to 'problem-solving'; Finally, students can reach a point were they decide that they have achieved their goals and are 'done'. Assuming the students get 'done' they will have some sort of product that they tacitly or explicitly believe addresses the goals of the mathematical task. If the process of interactive problem-solving is successful, this product will be a constellation of ideas, possibly including verbal, written, and/or graphical symbolic content whose meaning has been worked out through communication around the task. However, as analysis in thesis suggests, there are often times when this process breaks down or becomes distorted. This model can then help to diagnose what is at play and may provide insights into how to intervene to re-establish productive problem-solving small group interactions.

This model represents an ideal carefully constructed from classroom observations, professional background knowledge, analysis of transcripts of episodes of utterances, interviews, reflective memos from participant observation, and theoretical insights from the theory of communicative action. What is done in this thesis is an attempt to show how this model can be used to analyse various episodes of utterances to illuminate tacit features of the model and make sense of patterns of interaction in the small group interactions. Chapter 6 deals with the initial stages of using the theories to analyse episodes of utterances to establish potential productivity for analysis of the codes and model. This work addresses the first research question, seeking to demonstrate in a transparent fashion how small group interactions can be understood. Chapter 7 deals with examples of coordinating, problem-oriented constative, and validity-discourse interactions. The work of Chapter 7 uses the analysis in previous chapters to explore the patterns of interaction in the data, and reinforcing the productivity of the analytical framework for understanding small group interactions. Chapter 8 deals with examples of breakdowns in communication. And Chapter 9 deals with teacher intervention in support of small group communication, through strategic attempts to model communicative practices, and through the use of a pedagogy focused on establishing productive conditions for groupwork.

The analysis in this thesis lays the theoretical groundwork for exploring what the critical potential of these codes and theories may be. I argue that there is a way in which understanding small groupwork interactions in the manner set out in this thesis can contribute to a critical theory of education, rather than merely being a description of how things are in three classrooms from the perspective of one researcher. However, even if it were just the latter, then the contribution to the field would be as an example of how to characterise small group interactions using the linguistic formulations found in the Theory of Communicative Action. In this sense it contributes to a constellation of work in mathematics education including ideas about argumentation (e.g. Krummheuer 2000; Boero 2010), interaction (e.g. Cobb et al. 1992; Cobb and Bauersfeld 1995), and communication (e.g. Ongstad 2006; Sfard and Kieran 2001). Further, it considers the micro-analysis of students interacting in mixed ability mathematics classes using complex instruction, contributing to a constellation of work around ‘Complex Instruction’ in mathematics education (e.g. Boaler 2006; Boaler and Staples 2008; Staples 2008; Boaler et al 2010; Sebba et al. 2011). While these contributions are original and potentially valuable in themselves, the argument for the potential of this analysis for critique is a further claim to knowledge that addresses the third research question of this thesis: What is the critical potential of understanding student interactions from a perspective of communicative action?

10.2 The critical potential of understanding small group interactions from the perspective of Communicative Action

In order to consider the critical potential of this research, I need first to return to and expand upon some key issues of Critical Mathematics Education discussed earlier in Chapter 2 Sections 2.3. Skovsmose (1994), as one of the leading theorists of Critical Mathematics Education, has a role to play in the delimitation of the crisis in mathematics education, and he has a role to play in the search for a critical potential in understandings of mathematics education. In this section I closely treat a series of concepts from Skovsmose and then make an argument for the critical potential of the work in this thesis.

Critical education must address some crisis or crises. Crisis as defined by Skovsmose (1994) is about conflict. In the quote below he argues that conflict is a fact of life in modern society the existence of which does not need further justification.

> I can use terms like suppression, conflict, contradiction, misery, inequality, ecological devastation and exploitation; and it is impossible to deny the relevance of such terms. I shall use the term crisis to include all these phenomena, although I shall continue to use the other concepts as well. They refer to a semantical framework from which it is impossible to escape. (Skovsmose 1994, p14)

Yet even the need to say that there is no need to prove the existence of conflict in society suggests that conflict is somehow obscured in our day-to-day lives and in the popular imagination of the society that we inhabit. Skovsmose says that this obfuscation constitutes an ideology of the status-quo. This is a provocative statement which adds another element to a conceptualization of critical theory: 1) conflict; and, 2) obfuscation of conflict by a dominant narrative representing the interests of the status-quo.

> Ideologies hide or disguise conflicts and therefore they tend to reinforce established power structures of society. It has never been urgent for power centres to undermine ideologies which explain how the established social order in fact copies the intrinsic order of things. (Skovsmose 1994, p16)

The above articulation can be interpreted as an indictment of positivistic tendencies in policy and research to construe social reality as reflective not of a particular social reality situated in its historical context, but rather as a reflection of an objective reality. Conflating the distinct (yet connected) realms of objective reality and social reality is thus seen as inherently ideological and in the service of maintaining the status quo. The challenge for this thesis is whether it contributes to the kind of understanding that claims to reflect an established reality in a manner which merely props up the status quo, or whether there is some way in which understanding small group interactions from the point of view of communicative action can contribute to a critique of the status quo. Is there an ideology obfuscating conflict, which can be addressed through the theoretical developments of this research? So one needs to locate conflict, ideological obfuscation, and finally whether or not the research can contribute to a critique.

It would be helpful to clarify what is meant by critique, Skovsmose is there to help;

> A critique of ideology is directed towards certain belief systems and attempting to do this in a theoretically based and more organised way is what characterises a critical theory. The Frankfurt School indicates one possibility for realising this. According to the positivistic research paradigm, the only things possible to deal with in research are facts, and the objective of research is to identify correlations between different sets of facts. Research has only to do with what is actual. But that is not the only aspect involved in critical theory. To be critical means to be directed towards a critical situation and to look for alternatives, perhaps revealed by the situation itself. It means to try to identify possible alternatives. Positivistic research looks for what is actual; critical theory looks for what is possible in light of what is actual and what is critical. (ibid. p17)

So now one may consider the question: does the research in this thesis contribute to considering what is possible in light of what is actual? I would argue that the theoretical model articulated builds on what is actual and suggests possibilities that would address actual inequity. Take for instance the discussions of power and how it comes into play in actual episodes of utterances. The evidence of conflict in small group interactions can be analysed using the tools and frameworks developed in this thesis. So, having established what is actual (conflict around power in small group interactions), I now consider what is critical and what is possible. The critical has to do with the roles schools have as reproducing the conditions of society and in particular the crucial role that mathematics education plays in the establishment of hierarchal access to power within the system.

> This could be summarised by the thesis that schooling leads to the reproduction of social structures. This reproduction includes a reproduction of the division of labour, a reproduction of the distribution of power between the individual and the state and between social groups, and finally a reproduction of traditional cultural values. In short, the critical aspects of society are part of life in the school. A critical education must seek to respond to this. (ibid. p23-24)

So now what is possible given the actual and the critical? Can the research in this thesis contribute to mitigation of conflict and the establishment of more equitable social conditions? That seems ambitious. However, awareness and means of analysing the actual power conflicts and the actuality of inequitable power distributions within small group interactions suggests that this research may have a role to play beyond merely propping up the status quo. To see how this may be possible let us consider Boaler's comments on the relationship between equitable teaching practices and the production of conditions conducive to democracy:

> I contend, as have others (Cogan & Derricott, 1988; Steiner-Khamsi et al., 2002), that one of the goals of schools should be to produce citizens who treat each other with respect, who value the contributions of others with whom they interact, irrespective of their race, class or gender, and who act with a sense of justice in considering the needs of others in society. A first step towards producing citizens who act in such ways is the creation of classrooms in which students learn to act in such ways, for we know that students learn a lot more than subject knowledge in their school classrooms. (Boaler 2008, p167-168)

These comments are made in the context of analysis complex instruction approaches to mathematics education. Boaler identifies here a normative imperative for equitable approaches to the teaching of mathematics. However, as I have shown in this thesis, the use of equitable groupwork strategies for the teaching of mathematics does not automatically do away with the inequitable power relations that pervade the school mathematics context. This work implies a need for closer examination of what might be happening in complex instruction classrooms. The codes, models, and theory developed can contribute to that work (although it could also be pursued from different perspectives).

The actuality addressed by this research is the inequitable distribution of power and the existence of conflict in classrooms that are beginning to adopt complex instruction approaches to mixed ability year seven teaching. The critical context is the continuing reproduction of inequitable outcomes in England, especially in mathematics and sciences (Boaler, Altendorf, & Kent 2010) and the normative imperative of educating students in a manner conducive to participation in a democracy (Boaler 2008). The possible is addressed in this research through the identification of principles, which should be conducive to more equitable communicative interactions, based in analysis of evidence from actual episodes of utterances between participants. In short, this research may have a critical potential in that it points towards inequity and conflict in the actual and provides tools and models for thinking about how to approach understanding small groupwork from a more equitable perspective. For instance in the last chapter I began to articulate an interpretation of teacher action as a constellation of strategic and communicative action seeking to create opportunities for students to participate communicatively in the development of understanding in small group interactions in mathematics classes. Analysis such as this points towards how this work may be able to realise a critical potential through further work. Understanding how problems exist in the use of small group interactions from a communicative perspective and how participants seek to handle these problems can serve as the basis for attempts to resolve these problems and achieve greater equity and understanding in the use of groupwork in mathematics classes.

10.3 Practical contributions

The communicative perspective developed in this research allows participative access for researchers and educators to the development of meaning in small group interactions. Insights from the micro-analysis and models based in this perspective of communicative action can give educators pragmatic insights into curriculum design, principles and challenges in the structuring of groupwork and associated instruction, and a framework for understanding that may inform communicative intervention in the support of collaborative understanding. Considering the theoretical and critical challenge of trying to strategically act to create conditions for communicative action within the institution of school mathematics classrooms, this research has a practical contribution to make in providing insight into how to analyse small group communication in the classroom and potential approaches for creating more equitable and effective conditions for such communication.

Aside from task development, which could benefit from insights from this research around what kinds of task features are amenable to being addressed through communicative action, there is a practical benefit to be realised here in making problematic the use of mixed ability groupwork. This research highlights the challenges facing teachers and students in the adoption of Complex Instruction approaches to teaching mathematics in mixed ability year seven classrooms in England. Being aware of these challenges may empower teachers, teacher-educators, and researchers in meeting these challenges and addressing them in a critical fashion through thoughtful professional development, collaborative development of resources, and a framework with which to make sense of small group interactions.

10.4 Limitations

This thesis has several categories of limitations which are addressed in turn in this section, after a brief overview. Primarily among them are the limitations of a Case Study approach. Other limitations include aspects of the data: messy data, limitations based on negotiating access, ways in which working alongside a larger project created conflicting pressures, the snap-shot nature of data, researcher background/bias and limits of attempts at transparency in analysis. Also many important aspects of the social reality of the classrooms and participants were not addressed due to the limited focus of the research. These include, but are not limited to, ethnicity, gender, and socio-economic status. In general very little was done to address aspects of identity of participants.

I will begin with a discussion of fuzzy generalizations and the limitations of the claims made in this thesis in particular with regards to potential for generalization. Fuzzy propositions and fuzzy generalizations are set in contrast to scientific and statistical generalizations (Bassey 1999). They suggest that something may be the case, without attaching a measurement of its probability. Such fuzzy generalizations and propositions are useful as a form of empirical finding in that they can illustrate the potential significance of research to other practitioners. This is especially important in case study research as it is already explicitly a study of a singularity that holds no necessary correspondence to the larger population of related social situations that could serve as a basis for statistical or scientific generalization.

I have endeavoured to employ Bassey's model of case study research in the analysis of the data in this research, generating analytical statements at the different stages of the analysis and testing them and refining them in service of developing theories to understand relationship between equitable teaching approaches, student interactions and learning mathematics. However, the claims are necessarily particular and any use of ideas or findings in different context would require an interpretative effort that would necessarily alter the meaning of the findings from this research. Thus the most that can be hoped for from this research is that it may contribute in a meaningful way to future work. In particular it would be foolish to establish policy based on the findings in this research, although there may be a way in which this kind of case study would inform work that would be more appropriate for informing policy.

Negotiating access to classrooms and teachers' practice in the context of a larger project had benefits and drawbacks. The teachers were already adopting challenging new practices and committed to participating in research activities that added to their workload with no real compensation for their efforts. Further it was imperative to keep the work on the larger project distinct from the particular work on the research for this thesis. This presented challenges at times, as interesting data could not be used, as it was part of the wrong data sets, despite being relevant. In negotiating access I had real challenges as teachers pushed back against my initial plans to observe their classrooms intensively over a period of several weeks to a month. In the end I spent only a few days in each of the classrooms with the teachers and the students. However given the wealth of data generated by the flip cameras and the time necessary to make sense of dozens of hours of small group interactions in noisy classrooms, the amount of data collected was not a key limiting factor. Important aspects of identity such as socio-economic status of individual students, the performance of gender roles, and academic status, came up in the analysis of small group transcripts. The analysis would have been more able to address these with the benefit of a broader focus and more contextual data. If I were to do further research based on the work in this thesis I would want to include longitudinal tracking of students, student interviews and questionnaires as it could add to the kinds and quality of analysis possible.

This research was a learning process in which I came to realise that in order to more properly understand what is at play in small group interactions, micro-analytic models and theories must be complemented by research which moves beyond the classroom addressing the constellation of family, social, and institutional contexts in which these interactions take place. The microanalysis in this study illuminates the potential productivity of methodological and theoretical approaches developed and used in this research. This study is limited in many ways by the choices that I made about what to focus on and how to go about analysing and making meaning of the data. In particular the severe limits of the snapshot taken made it difficult to meaningfully interpret what was at play as power conflicts arose in the small group interactions.

Yet these limitations illuminate a potential path forwards in analysing social inequity in multiple facets and bringing these facets into relation with each other from a more sociological perspective as noted (see Chapter 9 Section 9.5). Future work might incorporate anthropological and ethnographic analysis of lifeworldy practices (complementing this research) done in conjunction with sociological surveys of institutional cultures and structures could be approached using constellations of critical social theory and drawing from existing empirical research to address the ongoing particularities of the persistent inequitable status quo in mathematics teaching and learning. Such an approach would be designed to take into account the limitations acknowledged here, that more needs to be known about the participants, the features of their particular lifeworld and lifeworldy experience over a longer period of time, and that this knowledge needs to be further complemented by an investigation of how these lifeworldy features relate to the broader systematic features in society. It is perhaps only with an integrated approach of this kind that the particular mechanisms of the reproduction of inequity through mathematics teaching and learning may be more fully understood, and through understanding begin to be overcome.

10.5 Further work

Further work based on this research includes plans for communication of findings through talks, poster presentations, and journal articles. Also I am already involved in developing networks of teachers interested in developing mixed ability approaches to teaching using complex instruction. There may be an opportunity to contribute to the development of curriculum resources, which this work could inform. Implications for practice include notably: the use that a communicative perspective may be to teachers and researchers in deciphering the complex multi-faceted issues of teaching; potential strategies for intervention and participation in small group and whole class discourse; the importance and nature of normative elements of classroom practice and culture; and finally the ways in which the rationality of mathematics can be related to and built upon the everyday rationality of communicative competence.

There is potential for use of the tools for microanalysis from a perspective of communicative action developed in this study in other studies. In particular these ideas might prove helpful in the analysis of data from the latest phase of the REALMS project. One might evaluate groupwork using the intersubjective model of student interaction in a number of ways: one could seek to examine correlations between patterns of utterances within the model and the sophistication of the products created, for instance. That is not done in this thesis and is mentioned merely as an example of how such a model might be used as a theoretical basis for analysis in future research. There is also potential for use of these ideas in relation to other sociological theories (as noted previously) as an avenue for exploring the connections between the micro-analysis of classroom speech acts and the wider social issues that form the context of such action proposed. This would be a significant contribution, but also very challenging in light of gap between the micro and the macro- I am already aware of my tendency to want to connect micro-analysis to wider social issues without necessarily considering what might be termed the necessary and sufficient evidence of micro-macro connections. It would also be really interesting to pursue an account of the relationship between subjectivity, identity and an intersubjective perspective based in communicative action as a future programme of work. There is potential in Habermas' use of concepts of intersubjectivity and the overlap with psychoanalytical approaches to develop a theory of identity formation in mathematics classes from a perspective of communicative action.

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Appendices

Appendix A: Example of Consent Letter

Date

Dear Parent/Guardian,

I am a Doctoral Student in mathematics education at the University of Sussex. I am conducting research for my thesis into the effectiveness of different teaching approaches in mathematics. I am conducting the study in a number of schools across the country; [School Name] has been selected as one of them and I will be visiting the school over the coming months.

In my visits I plan to observe and video mathematics lessons and to understand more about the students' interactions in mathematics lessons. The names of students, parents, teachers, administrators and schools will be kept completely confidential at all times. They will not be kept or released to anyone. Individual privacy will be maintained at all times.

The video recordings of lessons, which will also help me in my analysis, may be used in educational settings such as the training of teachers, professional conferences, research meetings and teacher education seminars.

One of the purposes of this letter is to introduce myself to you and describe the research project. The other is to tell you that your child's participation in the study is entirely voluntary. If, for whatever reason, you do not want your child to take part in the research or if you would like to discuss this research further please do not hesitate to contact me or [Teacher] at [School].

Thank you for your consideration,

Yours sincerely,

Geoffrey Kent

Doctoral Candidate in Mathematics Education
University of Sussex

Appendix B: Initial Codes

1	Action: articulating understanding of task
2	Action: following direction
3	Action: following direction within text
4	Action: orientation towards task resource
5	Action: redirecting resources
6	Action: relating discussion with teacher
7	Action: Taking control of process, delegating
8	Action: taking control of resources
9	Activity: filling out chart
10	Affirmation of response
11	Affirmation of strategy identified
12	Challenges appropriateness of response being recorded
	on basis that it is a question they have already answered
13	Challenging process: attempting to maintain order
	(first everyone responds to the second prompt)
14	Clarification statement: definition
15	Demand for control of activity
16	Directions: how to proceed with task
17	Following directions
18	Identification of activity required by task
19	Identification of instructions within text
20	Identification of prior confusion

21	Identification of prior ideas
22	Justification of action: appeal to authority of teacher
23	Justification of task articulation
24	Justification: Providing examples
25	Orientation towards task resource
26	Orienting question
27	Orienting statement
28	Question regarding definition of terms: orientation towards other's understanding
29	Question: clarification
30	Question: orientation to task resources, next steps
31	Question: others' understanding
32	Reading task out loud
33	Recording statement and checking for validity
34	Relating task to directions in worksheet
35	Reorientation of group activity on next prompt
36	Reorienting towards other's previous insight (unshared)
37	Reorients discussion away from process dispute t
38	Request for clarity of prior confusion
39	Request for next steps: implicit orientation of activity
40	Response to clarifying question
41	Response to probing question
42	Response to question: orientation towards task resources as tool
43	Response to request for clarity
44	Response to statement of Confusion: agreement
45	Response to statement of understanding; Expanding categories beyond examples

46	Response to teacher intervention
47	Response to Teacher, integrates ideas from discussion
48	Restating the task in relation to the resource
49	Role identification
50	Role question
51	Role statement
52	Statement of Confusion
53	Statement of prior activity phrased as a question
54	Statement of understanding
55	Statement of understanding; Identification of examples to support understanding
56	Statement: acceptance of justification
57	Statement: articulation of task
58	Statement: attempt to support validity challenge
59	Statement: Challenging validity of task articulation
60	Statement: definition focused on validity challenge
61	Statement: focused on validity challenge
62	Statement: focused on validity challenge, use of example to defend validity of original
	articulation of task
63	Statement: Identifying potential categories for investigation
64	Statement: justification of appropriateness of question recorded
65	Statement: poor process
66	Statement: Possible strategy, very detailed
67	Statement: Restating the task
68	Statement: strategy proposal; implicit

69	Statement: validity check directed at others
70	Statements of understanding (agreement)
71	Taking control of process: orientation à inclusion
72	Teacher intervention: affirmation of question recorded
73	Teacher intervention: clarification
74	Teacher intervention: direction
75	Teacher intervention: leading question regarding overlooked category
76	Teacher intervention: orientation à understanding goals
77	Teacher intervention: request for status update and implicit activity orientation.
78	Teacher whole class intervention: calling up one student from each group
79	Teacher: Affirmation of response
80	Teacher: Probing Question

Action		
Active(AA)	Discursive(AD)	Passive(AP)
Action: redirecting resources Action: Taking control of process, delegating Action: Taking control of process, directing Action: taking control of resources	-Action: articulating understanding of task -Action: supporting validity of claim through recourse to technical computation -Action: application of mathematical property	Action: following direction Activity: filling out chart Action: relating discussion with teacher Action: orientation towards task resource

Statement			
Problem Solving (Misc./Focused on task) (SPS)	Coordinating(SC)	Validity Claims: Identification and support(SV)	Validity challenges(SVCh)
Statement: applying mathematical property Statement: clarification of mathematical property Statement: conjecture regarding potential strategy Statement: mathematical property Statement: orientation implicitly identified pattern Statement: relating factors identified Statement: orientation filling out chart Statement: orientation identifying factors Statement: extension of conjecture to new case	-Statement: Demand for legitimate participation -Statement: denigrating response; taking control of process -Statement: orientation to teacher feedback -Statement: orienting group's work to class time constraints -Statement: positive encouragement orientation group goals -Statement: orientation working together on task -Statement: relating directions to group -Statement: suggestion for division of labour -Statement: support for proposed division of labour	Statement: clarification of claim Statement: Agreement with claim Statement: identifying alternative claim as valid Statement: support for s1's validity claim Statement: reversing validity claim Statement: supporting validity claim in s4's response, false Statement: supporting validity of previous statement	Statement: challenging mathematical claim Statement: challenging validity of conjecture Statement: challenging direction Statement: Challenging validity of abstraction Statement: Validity challenge Statement: supporting validity challenge Statement: reversal of previous validity challenge Statement: withdrawal of validity challenge

Question		
Coordinating(QC)	Discursive(QD)	Problem Solving (QPS)
-Orienting Question: roles -Question: focusing group on sharing out -Question: focusing group on teacher intervention	-Question: demand for reason, tacit validity challenge -Question: orientation towards others' understanding -Question: orientation validity challenge -Question: others' understanding -Question: reflective check of validity claim	-Question: Clarification, loaded -Question: clarification/probing -Question: orientation to task resources, clarification -Question: orientation identifying factors -Question: probing -Question: probing or clarification or both -Question: probing, exploratory -Question: probing, rhetorical -Question: strategic -Question: confusion? -Question: Identification of potential useful resource -Question: relating to prior work

Teacher Intervention		
Coordinating(TC)	Discursive: modelling problem-solving (TD)	Authoritative (TA)
-Teacher intervention: implicit activity orientation Teacher intervention: orientation towards sharing out	-Teacher intervention: maintaining discourse- restating question -Teacher intervention:	-Teacher intervention: deferring authority for mathematical content back to students; Probing question

-Teacher intervention: Orienting question, implicit direction -Teacher Intervention: suggestion orientation on inclusion, justification based in mathematical nature of tasks -Teacher whole class intervention: calling up one student from each group -Teacher intervention: direction -Teacher intervention: orientation à understanding goals -Teacher intervention: request for status update and implicit activity orientation	probing question -Teacher intervention: validity challenge -Teacher intervention: clarification -Teacher: Probing Question -Teacher intervention: orientation reflective problem-solving -Teacher intervention: leading question regarding overlooked category	-Teacher intervention: affirmation of response, orientation on collaborative problem-solving norms -Teacher intervention: affirmation of question recorded -Teacher: Affirmation of response

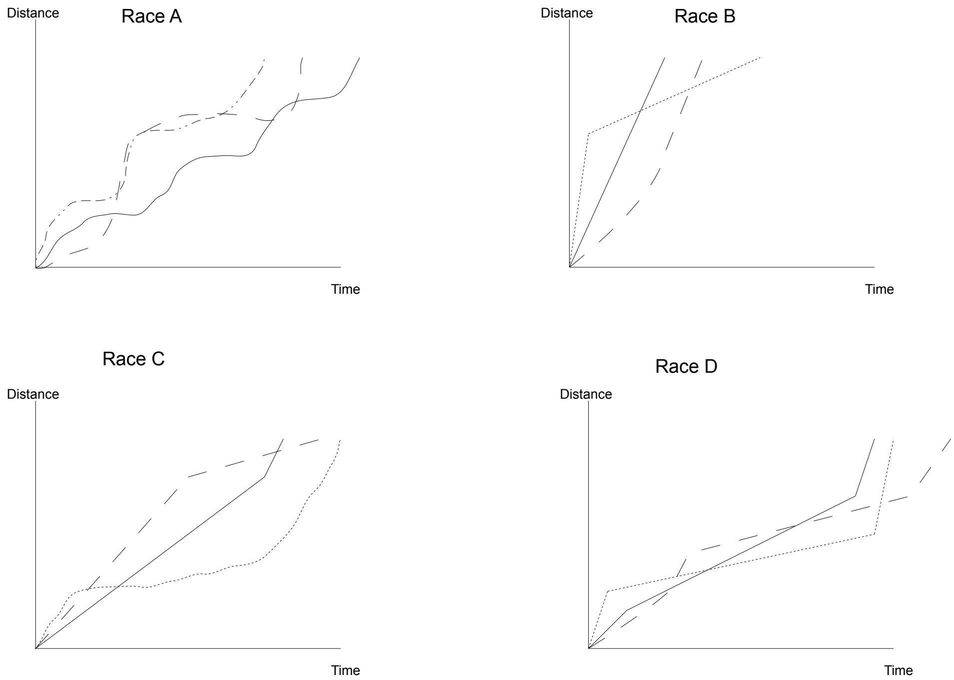
Response		
Coordinating(RC)	Problem Solving(RPS)	Discursive Validity(RD)
-Response to accusation: Objection -Response to normative threat: compliance -Response to question; Defence of status; implicit denigration of S3's academic status -Response to teacher intervention: agreement, hesitant -Response to teacher intervention: articulation of problem-solving -Response to teacher	-Response to clarifying question: explanation of work -Response to clarifying question; articulation of next steps -Response to probing question; articulation of reason -Response to question -Response to question: affirmative -Response to question; Question: orientation	-Response to implicit validity challenge: agreement -Response to question: justification of claim -Response to question: justification, correct -Response to question: justification, incorrect -Response to statement: challenging validity -Response to understanding check -Response to validity challenge -Response to Validity

intervention: description of (attempted)division of labour -Response to teacher intervention: orientation to implied directions -Response: defence of legitimacy of participation	identifying factors -Response: agreement -Response: attempt to add to mathematical property -Response: attempt to clarify and justify? -Response: conjecture; -Response: orientation identifying factors, false -Response: to clarifying question -Response: validating conjecture using previous mathematical property	challenge: defending generalization of mathematical property -Response to validity challenge: defence -Response to validity challenge: objection -Response to validity claims: agreement -Response: acknowledgement of clarification + drops challenge -Response: alternative claim -Response: claim -Response: support for alternative claim

Appendix C: Examples of Pattern Analysis Codes

Statement	**Code**
Status identities can interfere with intersubjective communication.	STAT_INT
Many utterances coded as problem-solving are better characterised as 'constative'.	CONST
Tacit aspects of the preconditions for intersubjective communication are essential for achieving understanding.	TACIT
Utterances can act on several levels simultaneously in intersubjective communication.	MULT
Confusion acts as a validity challenge in intersubjective communication.	CONF
The practice of intersubjective communication plays an important role in accessing cultural knowledge in a meaningful way.	CULT
Teacher intervention risks breaking down intersubjective communication by entering into an interaction as an authority.	BRK_AUTH_TI
Teacher intervention can contribute discursively to intersubjective communication.	CONT_DISC
Intersubjective communication can breakdown due to the recourse to authority of a participant in communication.	BRK_AUTH_S
An emphasis on justification is necessary for intersubjective communication to take place without being distorted by the recourse to external authority.	JUST
The tacit or explicit use of power distorts intersubjective communication.	PWR_DISTRT
Breakdown in intersubjective communication can be overcome through recourse to justification.	UNBRK_JUST
Teacher Intervention can Facilitate intersubjective communication.	CONT_TI
Status identities can act as motivation to engage in intersubjective communication.	CONT_STAT

Appendix D: Olympic Graphs Task



Olympic Graphs

Task

1. Your group is a team of commentators. Each person will be given a graph to look at which represents the athletes in a race. Working **individually** you must outline the key points in the graph, think about:
  - Who won the race?
  - Were they in the lead all the way?
  - Are there any points where the graph changes? What does this mean?
  - Can you make any comparisons between the different athletes?

   Using this information make a commentary to go with your graph.

2. Once you have considered your own graph you must look at all four graphs **as a group**, it is important that you make comparisons between them, noting any similarities or differences.

   Your challenge is to identify which graph is associated with which Olympic event.

You will be reporting back to the rest of the class on your findings so you must ensure that all members of your group are convinced by your choices and that you are able to give reasons for these.

Success Criteria

	Excellent	Good	Satisfactory	Poor
Knowledge and Understanding	All students in the group have used the information provided to fully understand the relationships between speed, distance and time and recognise how this can be represented using graphs.	Most students in the group are able to use the correct information to draw on the relationships shown in the graphs.	While some students in the group may have clear understanding of the relationships this is not communicated to the rest of the group.	Very little understanding of the task in the group and a lack of recognition of the relationships shown in the graphs.
Communication	Discuss all methods and results in detail and communicate these confidently to the class.	Discuss methods and results and be able to communicate these to the rest of the class.	Discuss some methods and results. Hesitance to feedback confidently to the rest of the class.	Very little discussion taken place and reluctance to feedback to the rest of the class.
Representation	Select the correct information required and use a variety of ways to represent findings.	Use the information provided to provide solutions and represent these in a way which is clear.	Use the information to represent some clear solutions.	Finds it difficult to select the necessary information and representations of solutions are unclear.
Reasoning	Confidently justify all ideas by making logical connections based on full understanding of the relationships shown in the graphs.	Justify most ideas by making connections based on the relationships shown in the graphs.	Justify some ideas by making connections based on the relationships shown in the graphs.	Very limited justification is used to make connections between the relationships shown.

In order for your group to achieve at this task the following areas must be considered:

Appendix E: Codes Task

a	1	n	14
b	2	o	15
c	3	p	16
d	4	q	17
e	5	r	18
f	6	s	19
g	7	t	20
h	8	u	21
I	9	v	22
j	10	w	23
k	11	x	24
l	12	y	25
m	13	z	26

Crack The Code!

At each table there will be two teams.

Each team will be given a rule to encode their message. Once they have encoded their message they will pass the encoded message to the other team to try to crack. Now this task might seem quite hard but your Bothan spies have found out two possible codes that your opponents may be using… But which code is the right code? Work with your team members to determine which code it is. When you have cracked the code come up with another set of two codes, choose one to encode your message, then send the message and the two codes to another group which is finished with their first code via the teacher. At the end of the class the teams will tally scores: 1 Point per cracked code.

Message conditions:

Messages should be no more than five words long. When you code the message you do not have to put spaces between the words. So your messagecouldlooklikethis beforeyou translate it into numbers and encode it.

Team Checklist

We have encoded a message with our partner using one of the rules

We have analyzed and decoded another teams message.

We developed our own code and encoded another message

We have analyzed and decoded another teams message from their own rule.

Group Discussion Questions:

How did you use the rule that encodes the message to decode the messages?

Can you come up with a rule to decode messages using a particular code? How?

Code 1 **2x+7** **Code 2** **14x-5**	**Code 1** **3x+6** **Code 2** **13x-4**
Code 1 **5x+9** **Code 2** **11x-2**	**Code 1** **4x+8** **Code 2** **12x-3**
Code 1 **6x+10** **Code 2** **10x-6**	**Code 1** **7x+2** **Code 2** **9x-7**
Code 1 **10x+4** **Code 2** **7x-9**	**Code 1** **9x+3** **Code 2** **8x-8**
Code 1 **8x+5** **Code 2** **6x-10**	**Code 1** **7x+6** **Code 2** **5x-11**

Success Criteria
In order for your group to achieve at this task the following areas must be considered:

	Excellent	**Good**	**Satisfactory**	**Poor**
Knowledge and Understanding	All students in the group have used the information provided to fully understand the relationships between coding and decoding using a given rule..	Most students in the group are able to use the correct information to recognise the relationships between coding and decoding using a given rule	While some students in the group may have clear understanding of the relationships this is not communicated to the rest of the group.	Very little understanding of the task in the group and a lack of recognition of the relationships between coding and decoding using a given rule
Communication	Discuss all methods and results in detail and communicate these confidently to the class.	Discuss methods and results and be able to communicate these to the rest of the class.	Discuss some methods and results. Hesitance to feedback confidently to the rest of the class.	Very little discussion taken place and reluctance to feedback to the rest of the class.
Interpreting and evaluation	Recognise patterns in your codes. Use appropriate procedures to form convincing arguments to support findings.	Recognises patterns and uses some appropriate procedures to form arguments to support findings.	Recognises patterns and begins to form arguments to support findings.	Finds it difficult to recognise patterns and form convincing arguments.
Reasoning	Confidently justify all ideas by making logical connections based on full understanding of the task.	Justify most ideas by making connections betweens different parts of the task.	Justify some ideas by making connections based on the relationships found in the task.	Very limited justification is used to make connections between different parts of the task.

Appendix F: Counting Cogs Task

Read this out loud in your group first!

Counting Cogs

Main Question: Which pairs of cogs let the colored tooth touch every tooth on the other cog? Which pairs do not let this happen? Why?

Task: Each member of your group will get two or three cogs to investigate.

1) Your task is to investigate the cogs you have and determine if they touch all of the teeth on the other cog with their colored tooth or not.

2) When you have determined that, record your work and trade one cog with another group member.

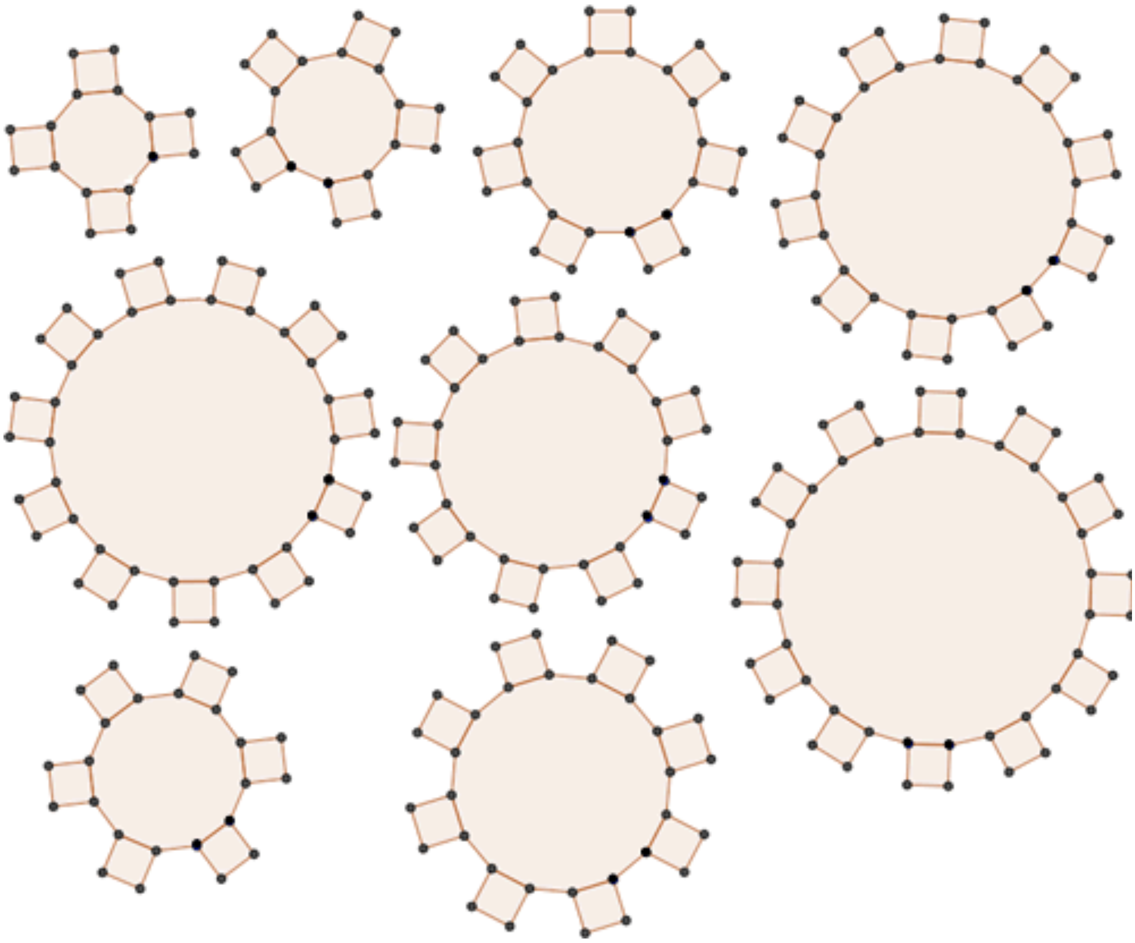
3) Investigate the new cog pair and repeat until all combinations of cogs have been investigated in your group.

4) Discuss any patterns you see with your group. What questions do you have? What conjectures can you make about why certain pairs work and others don't?

5) Record one question and one conjecture to share with the whole class.

Question:

Conjecture:



Appendix G: Counting Factors

Counting Factors

Recall that a prime number is an integer with exactly two factors, 1 and the number itself. In this activity we are going to investigate patterns of how many factors different kinds of integers have (not just prime numbers).

The number 1 is a factor of every integer. Every integer is a factor of itself. Therefore, every integer greater than 1 has at least two distinct factors and so must be prime or have more than two divisors.

Your task is figure out as much as you can about how many factors different kinds of numbers have.

- Investigate the numbers 1 to 100. Are there any patterns in the number of factors that different kinds of numbers have?

- What kinds of numbers have exactly 3 factors? What kinds of numbers have exactly four factors?

- Do larger numbers necessarily have more factors?

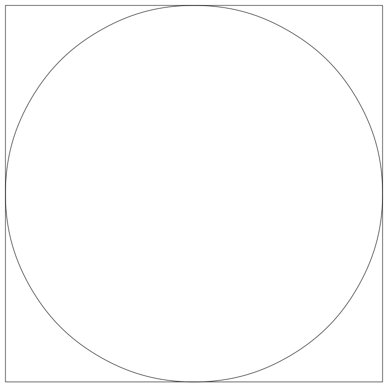
- Is there a way to figure out how many factors 1,000,000 has without listing them out and counting them?

In addition to answering these question come up with your own questions about how many factors different numbers have.

Describe any rules or patterns that you figure out.

Appendix H: Bracelets and Rings

Rings and Bracelets



A silversmith who makes jewellery needs to determine how much silver is needed to make rings and bracelets to fit different size boxes. The silversmith wants to make rings and bracelets that fit snugly into square boxes. The silversmith already has some square boxes of different sizes and wants to make rings and bracelets to fit into the boxes he already has, but he is not sure how much silver he will need for each ring or bracelet.

Can you find any patterns or rules to help the silversmith find how much silver he needs to make rings and bracelets to fit in any sized box?

Width of Bracelets and Rings	Depth of Bracelets and Rings
The bracelets are either ½ cm wide or 1 cm wide The rings are either 1 mm wide or 2 mm wide	The bracelets are either 1cm wide or 2cm wide The rings are either ½ cm wide or 3mm wide
Box Dimensions	**Cost of Silver**
The boxes range from 1.5 cm square to 10 cm square	1 cm^3 Silver weighs 10.5 grams Silver costs £10 per 50 grams

Width of Bracelets and Rings	Depth of Bracelets and Rings
The bracelets are either ½ cm wide or 1 cm wide The rings are either 1 mm wide or 2 mm wide	The bracelets are either 1cm wide or 2cm wide The rings are either ½ cm wide or 3mm wide
Box Dimensions	**Cost of Silver**
The boxes range from 1.5 cm square to 10 cm square	1 cm^3 Silver weighs 10.5 grams Silver costs £10 per 50 grams

Appendix I: Euclidean Algorithm (or Greatest Common Divisor task)

Euclidean Algorithm for finding Greatest Common Divisor (GCD)

Take any two numbers, for instance 1518 and 792. Then Subtract the smaller from the bigger number and record both numbers. Continue doing this until one number is reduced to zero. The number left is the GCD of the two numbers. Why?

First Number	Work	Second Number	Work
1518		792	
726	1518-792=726	792	
726		66	792-726=66
660	726-66=660	66	
594	660-66=594	66	
528	594-66=528	66	
462	528-66=462	66	
396	462-66=396	66	
330	330-66=330	66	
264	330-66=264	66	
198	264-66=198	66	
132	198-66=132	66	
66	132-66=66	66	
66		0	66-66=0

The number which is left, in this case 66 is the largest number that divides into both numbers with no remainder, and is called the Greatest Common Divisor.

Main question: Why does this algorithm, or set of rules, always give the Greatest Common Divisor?

Task:

1) Try using the algorithm several times to find the GCD of several pairs of numbers.
2) Look at your work and make some conjectures, or educated guesses, about why this set of rules works. Record your guesses about the algorithm's justification.
3) Look at your work and your conjectures and think of one question that would help you understand the algorithm and it's justification better. Write this question down.

Appendix J: Example transcript excerpt with codes

Harry goes on reading the task card until 3:11

Harry: ok does everyone understand? [Question: others' understanding] [QD; QC]

Thomas: we understand- now we want to move on [Statement of understanding] [SD/SC]

Harry: ok you understand [pointing at Thomas]; do you understand [pointing at Charlotte], Charlotte– off in a different world [Checking understanding; accusation of non-participation] [QD/SC]

Charlotte: no I'm not [Response to accusation: Objection] [RC]

Harry: ok Daniel do you understand? [Question: others' understanding] [QD; QC]

Daniel : yes I do [enthusiastically] [Response to question: affirmative] [RD; RC]

Thomas: I reckon we should divide into two so some people work on this and some people work on the other question [Statement: suggestion for division of labor] [SC]

Harry: This is a bit embarrassing Dan - ok I need a pen – ok you're sure – ok investigate the number of factors different kinds of numbers have – ok so Rafael [Orientation Statement] [SC]

Thomas: You and Daniel are going to fill out this [holding up the factor chart] and try and get as far as you can; it's very simple does everyone understand what a factor is? [Taking control of process: Division of labor; Statement: denigrating task difficulty; Question: others' understanding] [AA; QD; QC]

Charlotte: numbers that go into [Response to understanding check] [RD; RC]

Thomas: no it's numbers that multiply together to ; so one… let's just do the first ten [Statement: denigrating response; taking control of process] [SVCh; AA]

Harry: ok [SC]

Harry: ok the factors of number 1 are 1 and that's just one [Activity: identifying factors] [SPS]

Charlotte: ok 2 is 2 and 1 [Activity: identifying factors] [SPS]

[some discussion about the camera + embarrassment]

Charlotte: number 2 [Activity: identifying factors] [SPS]

Harry: 2 is 1 and 2; 1,2,3…1,2,3,4 [Activity: identifying factors] [SPS]

Daniel : aw this is gonna go on for ever [Objection to approach; expression of frustration] [SD]

Harry: 1,2,3,4,5 ? [Activity: identifying factors; mocking tone] [SPS; RD]

Charlotte: no – you have to think seriously about it….[Statement: Demand for legitimate participation] [RD]

Harry: what? -these are all the numbers that can go into …. [Response: defense of legitimacy of participation] [RD]

Thomas: just these two questions – we need to think about [orientating statement] [SC]

Harry: so just these two questions- but which problem? [grabs task card looks at it] the problem – [Question: Clarification] [QPS]

Thomas: that one [pointing] [Response to clarifying question] [RPS]

Harry: oh these [gesturing] [Acknowledgement of response] [RPS]